Heterogenous Forecasting and Federal Reserve Information

by

Mordecai Kurz, Stanford University
(This draft: December 19, 2001)

Abstract

We provide evidence that private forecasters and the staff of the Federal Reserve use different forecasting models to predict inflation and GNP growth and heterogeneity of forecasting models is the norm in the market place. We thus argue that neither the Fed nor commercial forecasters know the "true" model of the economy. We demonstrate that their forecast errors are correlated with information available at the time when the forecasts were made and hence, by studying the systematic patterns of these forecast errors, we can deduce a great deal about their assessments of economic conditions in general and their view of monetary policy in particular. We also show that

(i) although all private forecasters have the same information, their diverse forecasting models result in a *distribution* of forecasts which fluctuates over time. The distribution shows no tendency for convergence. The "consensus" median forecaster is a random member whose identity changes over time;

(ii) the evidence shows that at any date there is a whole set of private forecasts who *agree with the Fed's forecasts* but the Fed forecasts are less volatile than the volatility of private forecasts measured by the variance of the cross-sectional distribution of private forecasts;

(iv) diverse assessments of the impact of monetary policy on inflation and growth are important factors which contribute to the heterogeneity of forecasts. A surprising result reveals that although there is strong evidence for heterogeneity among forecasters, we also find similarity in *qualitative* patterns of forecast errors of all participants, including the Fed. Qualitative similarity implies similarity in the basic ideas underlying the forecasting models *even when these ideas are wrong*. This implies a degree of correlation among the subjective beliefs of divergent agents in the economy.

(v) although all forecasts violate the standard orthogonality conditions of Rational Expectations, we argue in this paper that this should not be interpreted as *irrational* behavior. We provide an introduction to the theory of Rational Belief (see Kurz (1994), (1997)) and demonstrate that in contrast to the rejection of Rational Expectations, the pattern of estimated parameters is consistent with the predictions of the theory of Rational Beliefs (see Kurz (1994), (1997)). In opposition to some (e.g. De Bondt and Thaler (1985), (1990)), we argue that rejection of rational expectations should not be interpreted to imply irrational behavior.

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1. Introduction

In a stimulating paper Romer and Romer (2000) (in short R&R) propose that the Federal Reserve staff has superior information in making their Green Book forecasts for the Open Market Committee. The method employed by R&R is familiar from the literature on forecast rationality. To explain it let $\pi_{t,h}$ be the annualized inflation rate and $g_{t,h}$ be the real annualized growth rate of GNP *h quarters after date t*. Denote the forecasts of ($\pi_{t,h}$, $g_{t,h}$) by agent k at date t, given information I_t by $\pi^k_{t,h} \equiv E^k[\pi_{t,h}|I_t]$ and $g^k_{t,h} \equiv E^k[g_{t,h}|I_t]$. Now suppose a sequence of forecasts of agent k for dates t = 1, 2, ... , T are publicly available and the measure of inflation the agent forecasted is the GNP deflator. Also, the Green Book forecasts of the Federal Reserve staff are available for the same dates and for the same forecasting horizons h. Using inflation forecasting as an example, R&R (2000) estimate first a regression of the form

(1a) $$\pi_{t,h} = \alpha_0^{h,k} + \alpha_1^{h,k}\pi^k_{t,h} + \varepsilon^k_{t,h}.$$

If the agent knew the true probability law of the stochastic process of inflation and if he used all available information, we would expect $\alpha_0^{h,k} = 0$ and $\alpha_1^{h,k} = 1$, conditions which R&R (2000) and others call "rationality conditions." Next, suppose we ask if the Federal Reserve staff knew something that the private forecaster did not know. Keep in mind that private forecasts are known to the Federal Reserve while the Fed's Green Book forecasts are kept confidential for five

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years after they are made. Hence, the Fed's forecasts *are not available* to forecaster k when he makes his forecasts. Under these conditions we may ask if the green book forecasts, denoted by $\pi^{GB}_{t,h}$, improve regression (1a). R&R (2000) thus estimate a second regression of the form

(1b) $$\pi_{t,h} = \beta^{h,k}_0 + \beta^{h,k}_1 \pi^k_{t,h} + \beta^{h,k}_2 \pi^{GB}_{t,h} + \epsilon^k_{t,h}.$$

They propose that, under the specified conditions, $\beta^{h,k}_2$ measure the impact of additional information the Federal Reserve Staff had relative to agent k, and statistical significance of these estimates demonstrate the presence of superior information of the Federal Reserve staff.

To carry out this analysis R&R (2000) used the Green Book forecasts from 1965:11 through 1991:11 and three private data files. First, the forecasts of the Blue Chip Economic Indicators (BLU) consisting of over 50 large corporations, financial institutions and consulting firms engaged in forecasting. These forecasts were made from 1980:1 through 1991:11 and cover different economic indicators. The second set is from the Survey of Professional Forecasters (SPF) which is currently conducted by the Federal Reserve Bank of Philadelphia. This survey continues the earlier American Statistical Association\National Bureau survey which was started in 1968. The SPF is similar to the Blue Chip Economic Indicators in being based on many different professional private forecasters. One difference between the SPF and the Blue Chip Economic Indicators is that the identity of the SPF forecasters is not revealed by name while the identity of the Blue Chip forecasters is made public every month. The third source are the forecasts made by Data Resources, Inc. (DRI) starting in 1970:7. Hence, among the three sources used, only the DRI data consist of time series of *a specific agent or organization*. The BLU and SPF files consist of heterogenous collection of forecasts. R&R (2000) used *the median forecast of BLU and the median forecast of SPF as if it was the forecast made by an individual agent.* The analysis is then conducted as if we have consistent forecast data of three agents.

To illustrate the results of R&R (2000) consider, for example a forecasting horizon of 3 quarters, thus h = 3. The upper panel of Table 1 provides estimates of equation (1a) and the lower panel estimates of (1b). These estimates are based on our enlarged data set which covers the period up to 1995, but are similar to those reported in R&R (2000). The estimated values of $\beta^{2,k}_2$ are significantly different from zero in all three equations and the correlation coefficients estimated for all equations in the second set are larger than the corresponding coefficients in the first set. Gaps in data sources limit the dates for which data are *simultaneously* available from multiple sources. Consequently, the periods for which the equations can be estimated and the number of useable observations depends upon the number of sources used. In the case of Table 1 the number of observations used in the upper panel is higher than in the lower panel.

Table 1: Test of Federal Reserve Staff Added Information

(standard errors in parentheses)

$$\text{Equation (1a):}\quad \pi_{t,3} = \alpha_0^{3,k} + \alpha_1^{3,k}\pi_{t,3}^{k} + \varepsilon_{t,3}^{k}$$

k	$\alpha_0^{2,k}$	$\alpha_1^{2,k}$	R^2	N
Blue Chip	- .38 (.40)	.86 (.11).	.71	243
SPF	.23 (.52)	.96 (.11)	.43	126
DRI	.82 (.46)	.82 (.10)	.47	248
Federal Reserve Staff	.29 (.38)	1.04 (.08)	.54	239

$$\text{Equation (1b):}\quad \pi_{t,3} = \beta_0^{3,k} + \beta_1^{3,k}\pi_{t,3}^{k} + \beta_2^{3,k}\pi_{t,3}^{GB} + \epsilon_{t,3}^{k}$$

k	$\beta_0^{2,k}$	$\beta_1^{2,k}$	$\beta_2^{2,k}$	R^2	N
Blue Chip	.13 (.28)	- .28 (.24)	1.15 (.25)	.84	129
SPF	.93 (.45)	- .71 (.25)	1.65 (.24)	.58	85
DRI	-.09 (.36)	- .20 (.22)	1.26 (.20)	.57	180

R&R (2000) conclusion of superior information of the Federal Reserve staff is deduced from the assumption that the Fed and commercial forecasters use the "true", rational expectations, forecasting functions *which satisfy orthogonality conditions with all information available at the time*. This assumption links the discussion to the extensive literature on forecasting rationality (e.g. De Bondt and Thaler (1985), (1990), DeLeeuw and McKelvey (1981), (1984), Frankel and Froot (1987), Friedman (1980), Keane and Runkle (1990), (1998), Leonard (1982), Lovell (1986) and Zarnowitz (1985)). Indeed, the SPF data discussed above was used by Zarnowitz (1985) to show that SPF forecasts violate rational expectations while Keane and Runkle (1990) used the same data to show that price forecasting of these professional forecasters are "rational."

In this paper we present evidence that the conditions of forecast orthogonality are strongly violated, implying that the R&R (2000) conclusion of superior Federal Reserve information is flawed. The violations of orthogonality hold for both the Federal Reserve staff as well as for private forecasters. Instead, we present strong evidence in support of an alternative perspective. This perspective is built on the evidence that private forecasters and the staff of the Federal Reserve *use different forecasting models to predict inflation and GNP growth* and heterogeneity of forecasting models is the norm in the market place. By implication we argue that neither the Fed nor commercial forecasters know the "true" model of the economy. Their forecast errors over short time spans are correlated with information available at the time when the forecasts were made and hence, by studying the systematic patterns of these forecast errors, we can deduce a great deal about their assessments of economic conditions in general and their view of monetary policy in particular. We also argue in this paper that

(i) although all private forecasters have the same information, their diverse forecasting models result in a *distribution* of forecasts which fluctuates over time. The distribution shows no tendency for convergence. The median forecaster is thus a random member whose identity changes over time;

(ii) the evidence shows that at any date there is a whole set of private forecasts who *agree with the Fed's forecasts*;

(iii) the Fed' model generates, over time, less volatile forecasts than the volatility of private forecasts measured by the variance of the cross-sectional distribution of private forecasts;

(iv) the diverse assessments of the impact of monetary policy on inflation and growth are important factors contributing to the heterogeneity of forecasts. A surprising result reveals that although there is strong evidence for heterogeneity among forecasters, we also find similarity in *qualitative* patterns of forecast errors of all participants, including the Fed. Qualitative similarity of forecast errors does not mean they make the same *quantitative* forecasts; it does imply similarity in the basic ideas underlying the forecasting models *even when these ideas are wrong*. This implies a degree of correlation among the subjective beliefs of divergent agents in the economy.

(v) violation of orthogonality of forecasts is inconsistent with Rational Expectations but this should not be interpreted as *irrational* behavior. We show that the pattern of estimated parameters is consistent with the predictions of the theory of Rational Beliefs (see Kurz (1994), (1997)). In opposition to some (e.g. De Bondt and Thaler (1985), (1990)), we argue that rejection of Rational Expectations should not be interpreted as irrational behavior.

2. The Data

2.1 *The Median Forecast Data*

We have noted that the data used in the regression studies consist of median forecasts and realized data of inflation rates (measured by the GNP deflator) and of real GNP growth rates. The detailed description of these data can be found in R&R (2000) pages 430-433. The R&R (2000) files cover the period up to 1991:11. We received this file from Christina and David Romer and at that time the Green Book forecasts were available up to 1995:11. We thus updated the R&R (2000) files to 1995:11 and this provided us with four additional years of data. The BLU and SPF surveys consist of forecasts of a large number of *different forecasters* over many years and this heterogeneity, central to our work, will be discussed later. As for the comparability of the data due to different dates of release, there are various problems which R&R (2000) discuss in detail. Without repeating this detailed description we make several comments that could help the reader understand the nature of the data used here.

(i) *Frequency of Data*. Different data sources are issued at different dates. The Green Book has been prepared by the staff for the Open Market Committee since 1965:11. In the 1960's and 1970's the committee met almost each month. Since the 1980's it has typically met eight times a year. Since there are no forecasts in months when the committee does not meet, the frequency of the Fed's forecasts has changed with the frequency of the committee's meetings. The Blue Chip Economic Indicators (BLU) are circulated to subscribers monthly around the fifth of *each month* starting with 1980:1. The forecasts of Data Resources, Inc. are issued *three times at each quarter*: one early, one in the middle and one late in the quarter since 1970:7 but the middle forecasts are available only since the first quarter of 1980. The Survey of Professional Forecasters (SPF) is conducted near the end of the second month of each quarter and all SPF data is quarterly. To attain maximal matching of data sources our forecasting files are arranged so that, for every forecast horizon, the data is constructed *as monthly data with missing observations*. The BLU data are frequent but cover only the period since 1980:1. The SPF forecasts for inflation are available since 1968:11 and for real GNP growth since 1981:8 but the main limitation imposed by SPF is the fact that it is available only four times each year.

(ii) *Forecast Horizon*. It is typically the case that at any date the actual inflation rate and growth rate of GNP are not known for the quarter in which the forecasts are made. Hence, each one of the forecasts includes forecasting for the "current quarter" and this horizon is denoted by h = 0. Hence, h = 1 means "the quarter *following* the quarter in which the forecasts were made." The Green Book horizon goes up to seven quarters into the future but varies over time. The BLU *consensus* forecasts (i.e. the medians of the individual distributions) are available for six and sometimes for seven future quarters. The DRI forecast horizon is typically seven future quarters and the SPF is typically four future quarters. The number of observations available for the long horizon forecasts is typically small and equations estimated for long horizon from multiple sources are very unreliable. In most of the paper we avoid the seven quarter horizon models due to insufficient data.

(iii) *Actual data on inflation and GNP growth*. Data revision has been discussed in the literature as a complicating factor since it raises the question of what forecasters are forecasting[2]. The initial GNP statistics, released about 45 days after the end of each quarter, are incomplete. These initial estimates contain significant errors since some component series are not available, hence revisions are needed. The first revision is released at the end of the *second* month following each quarter and the second revision at the end of the *third* month following each quarter (that is, at the end of the subsequent quarter). Further revisions are made each July and reevaluation each five years. We study in this paper all forecasting horizons hence we need a reasonably complete measure of inflation and growth which is conceptually close to what the agents were forecasting. We use the data released at the second revision: it is complete but is free of any conceptual reworking. It is thus conceptually close to what the agents are forecasting[3].

(iv) *Serial Correlation*. The presence of serial correlation in the forecast errors is inevitable (for details, see R&R (2000) page 433) and this suggests that a correction is in order. We use the Newey and West (1987) procedure to compute robust standard errors in all equations of this paper. This is an appropriate procedure which ensures positive variances.

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[2] For example, Zarnowitz (1985) used revised data to show that SPF forecasts lead to a rejection of Rational Expectations. Keane and Runkle (1990) objected and argued SPF provide forecasts of the first release not the revisions. Using the first release data they claim to have demonstrated that for a very short horizon, forecasts are compatible with Rational Expectations. We discuss their claim later, in Section 6.

[3] The R&R (2000) file stops in 1991:11 partly because they did not have the Green Book data and partly because of the switch from GNP to GDP in government statistics. In extending the data to 1995:11 we have elected to accept the possible distortion that might be created by this data switch. We think it is minor in relation to the forecast errors analyzed in this paper.

2.2 *Distributions of BLU and SPF Forecast Data*

An important additional data which we use in this study are *the panel data of individual forecasts* of inflation rates and real GNP growth rates reported by SPF and BLU. The SPF data is available in electronic form from the Federal Reserve Bank of Philadelphia[4]. The Survey assigns a number to every participating forecaster. Over time, forecasters drop out of the survey and new forecasters are added and a record is available for variations in the identity of the participants. The distributional data of the SPF is exactly as reported above: four forecasts each year with forecast horizons which include the current quarter in addition to four future quarters. The number of forecasters varies over time. The situation with BLU is different.

BLU reports each month the median or "consensus" forecasts for the current quarter and for six or seven quarter into the future. In addition it discloses the detailed list of forecasters identified by name, providing forecasts of various economic variables for the "Current Year" and for "Next Year." In other words, we have monthly data on forecasts, by some 50 corporations, financial institutions and consulting firms, of measures of future aggregate economic performance. The measures forecasted by the participants cover two full *calendar years*: one is the year in which the forecasts are made and the second is the calendar year which follows. The implication is that in any year we get 12 pairs of different forecasts of various economic variables for *the same* two full calendar years. These individual forecasts cover the period from 1980:1 through 2001:3.

3. The Fed's Forecasts Violate Orthogonality Conditions

In this section we present two arguments to show that the *Fed's forecasts violate standard orthogonality conditions* assumed by R&R (2000). In doing so we also set the stage for the later developments in section 5.

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[4] We have used the version of these files compiled by Dean Croushore of the Federal Reserve Bank of Philadelphia.

3.1 *Private Forecasts Improve the Performance of the Fed's Forecasts*

The hypothesis proposed by R&R (2000) is that the Fed has superior information *about the economy*. However, when members of the staff prepare their forecasts they know the forecasts of the private forecasters in our files. Hence, under the rational expectations assumption of R&R (2000), private forecasts should be orthogonal to the forecast errors of the Fed and contribute absolutely nothing to the statistical accuracy of the Fed forecasts. Also, any other information which is known to have been available to the Fed's staff should not improve the accuracy of the Fed's forecasts. We shall see that the data rejects this conclusion. Recalling that $g_{t,h}$ is the growth rate of GNP h quarters after date t, the main argument of this section is that the Fed's forecasting equations violate orthogonality conditions. Information which is known to have been available to the Fed will be shown to be correlated with the forecast errors of the Fed and improve the Fed's forecasts measured by regressions of the following type

(2a) $$\pi_{t,h} = \alpha_0^{h,GB} + \alpha_1^{h,GB}\pi_{t,h}^{GB} + \varepsilon_{t,h}^{GB}$$

(2b) $$g_{t,h} = \delta_0^{h,GB} + \delta_1^{h,GB} g_{t,h}^{GB} + \vartheta_{t,h}^{GB}.$$

To demonstrate this we start by estimating the parameters of (2a)-(2b) using our enlarged data files for 1965:11 - 1995:11. These estimates are the basic reference and Table 2 summarizes the results. Observe that the quarterly forecasts of GNP growth, beyond *one quarter into the future*, are of extremely low quality. This fact is true of both the Fed's forecasts as well as of private forecasts (see Table 6 below).

Table 2: Accuracy of Fed's Green Book Forecasts, Estimates of (2a)-(2b)
(standard errors in parentheses)

	Inflation Forecasts				*GNP Growth Forecasts*			
h	$\alpha_0^{h,GB}$	$\alpha_1^{h,GB}$	R^2	N	$\delta_0^{h,GB}$	$\delta_1^{h,GB}$	R^2	N
0	.35 (.22)	.97 (.04)	.83	294	.80 (.30)	.89 (.08)	.53	293
1	.36 (.31)	1.00 (.07)	.72	278	.88 (.54)	.77 (.13)	.25	277
2	.32 (.36)	1.03 (.08)	.60	256	.70 (.67)	.79 (.18)	.18	255
3	.29 (.38)	1.04 (.08)	.54	239	1.05 (.93)	.62 (.27)	.08	238
4	- .13 (.41)	1.09 (.09)	.54	209	- .15 (1.07)	1.08 (.33)	.16	208
5	- .36 (.42)	1.08 (.10)	.54	150	- .70 (1.15)	1.31 (.41)	.18	149
6	- .25 (.45)	.95 (.14)	.59	90	- .80 (1.11)	1.47 (.41)	.22	89
7	- .09 (.56)	.82 (.19)	.64	59	- .26 (1.93)	1.35 (.71)	.10	58

The rational expectations assumptions made by R&R (2000) implies $\alpha_0^{h,GB} = \delta_0^{h,GB} = 0$ and $\alpha_1^{h,GB} = \delta_1^{h,GB} = 1$ for all horizons h. We observe that although many point estimates of these parameters are not close to the specified values, given the standard errors of the estimates, *the data does not reject the hypothesis that almost all the Fed's parameters reported in Table 2 satisfy these "rationality" conditions.*

We consider now the contribution of private forecasts to the forecast error of the Fed's staff forecasts in the sense of improving the estimates of equations like (2a)-(2b). For example, we would like to include all three private sources and estimate equations of the form

(3a) $\pi_{t,h} = \alpha_0^{h,GB} + \alpha_1^{h,GB}\pi_{t,h}^{GB} + \alpha_2^{h,1}[\pi_{t,h}^{GB} - \pi_{t,h}^{BLU}] + \alpha_3^{h,2}[\pi_{t,h}^{GB} - \pi_{t,h}^{SPF}] + \alpha_4^{h,3}[\pi_{t,h}^{GB} - \pi_{t,h}^{DRI}] + \varepsilon_{t,h}^{GB}$

(3b) $g_{t,h} = \delta_0^{h,GB} + \delta_1^{h,GB} g_{t,h}^{GB} + \delta_2^{h,1}[g_{t,h}^{GB} - g_{t,h}^{BLU}] + \delta_3^{h,2}[g_{t,h}^{GB} - g_{t,h}^{SPF}] + \delta_4^{h,3}[g_{t,h}^{GB} - g_{t,h}^{DRI}] + \vartheta_{t,h}^{GB}$.

Under the assumption of superior Fed information, the null hypothesis is $\alpha_j^{h,k} = 0$, $\delta_j^{h,k} = 0$, j = 2, 3, 4. Unfortunately, the number of useable observations in estimating equations (3a)-(3b), for the combination of *all three private sources*, is only 42 for inflation and 44 for GNP growth and hence the estimates of ($\alpha_j^{h,k}$, $\delta_j^{h,k}$) for j = 2, 3, 4 would not be reliable. To increase the number of observations we examine the joint contribution of only BLU and DRI forecasts to the Green Book forecasts from 1980:1 through 1995:11. We thus estimate equations of the form

(4a) $\pi_{t,h} = \tilde{\alpha}_0^{h,GB} + \tilde{\alpha}_1^{h,GB}\pi_{t,h}^{GB} + \tilde{\alpha}_2^{h,1}[\pi_{t,h}^{GB} - \pi_{t,h}^{BLU}] + \tilde{\alpha}_4^{h,3}[\pi_{t,h}^{GB} - \pi_{t,h}^{DRI}] + \tilde{\varepsilon}_{t,h}^{GB}$

(4b) $g_{t,h} = \tilde{\delta}_0^{h,GB} + \tilde{\delta}_1^{h,GB} g_{t,h}^{GB} + \tilde{\delta}_2^{h,1}[g_{t,h}^{GB} - g_{t,h}^{BLU}] + \tilde{\delta}_4^{h,3}[g_{t,h}^{GB} - g_{t,h}^{DRI}] + \tilde{\vartheta}_{t,h}^{GB}$

for h = 0, 1, ..., 6. We exclude horizon h = 7 for which we have less than 40 observations. The results are reported in Table 3. We indicate by (*) any estimate of ($\alpha_j^{h,k}$, $\delta_j^{h,k}$) for j = 2, 4, which is statistically significantly different from 0 at confidence level which is *higher than 10%.* Observe that for 10 out of the 14 equations in Table 3 there are some estimates of ($\alpha_j^{h,k}$, $\delta_j^{h,k}$) for j = 2 , 4 which are significantly different from zero. Indeed, for *any combination* of private forecasters there are some horizons for which private forecasts improve, in a statistically significant manner, the forecast error of the Green Book forecasts as estimated in (2a)-(2b).

These results, combined with similar results reported in R&R (2000),[5] demonstrate that *private forecasts are correlated with the Fed's forecast error and contribute significantly to the*

[5] R&R (2000) report similar tests for inflation in their Tables 4 and for GNP Growth in their Table 6. For some reason they ignore the fact that their assumptions imply $\alpha_j^{h,k} = 0$, $\delta_j^{h,k} = 0$ for j = 2, 3, 4. Their Table 4 reports that 10 out of 11 equations contain estimates of parameters like these which are statistically significantly different from zero and Table 6 reports 7 out of 20 such equations. Their claim of superior Fed's information is deduced from the satisfaction of orthogonality conditions which are violated by their own estimates but ignored by them.

accuracy of the Fed forecasts. Hence, according to R&R (2000), one must conclude that private forecasters had some private information that was not available to the Federal Reserve staff when they prepared their forecasts. We know this conclusion to be false, the Fed's forecasts violate orthogonality and the conclusion of Fed's superior information may be questioned.

Table 3: Test of Fed's Forecast Error Orthogonality Using BLU and DRI Forecasts

(standard errors in parentheses)

Inflation: $\pi_{t,h} = \tilde{\alpha}_0^{h,GB} + \tilde{\alpha}_1^{h,GB}\pi_{t,h}^{GB} + \tilde{\alpha}_2^{h,1}[\pi_{t,h}^{GB} - \pi_{t,h}^{BLU}] + \tilde{\alpha}_4^{h,3}[\pi_{t,h}^{GB} - \pi_{t,h}^{DRI}] + \tilde{\varepsilon}_{t,h}^{GB}$

h	$\tilde{\alpha}_0^{h,GB}$	$\tilde{\alpha}_1^{h,GB}$	$\tilde{\alpha}_2^{h,1}$	$\tilde{\alpha}_4^{h,3}$	R^2	N
0	- .37 (.21)	1.01 (.04)	- .50* (.11)	- .39* (.11)	.93	116
1	- .16 (.31)	.94 (.06)	- .08 (.21)	.04 (.14)	.89	116
2	.21 (.31)	.88 (.06)	.46* (.20)	- .27* (.17)	.86	116
3	.63 (.29)	.78 (.06)	.68* (.24)	- .43* (.16)	.78	116
4	1.38 (.40)	.57 (.09)	.69* (.25)	- .57* (.24)	.70	112
5	1.51 (.45)	.48 (.09)	.68* (.31)	- .58* (.20)	.68	83
6	1.80 (.59)	.35 (.10)	.66* (.40)	- .69* (.27)	.52	48

GNP Growth: $g_{t,h} = \tilde{\delta}_0^{h,GB} + \tilde{\delta}_1^{h,GB} g_{t,h}^{GB} + \tilde{\delta}_2^{h,1}[g_{t,h}^{GB} - g_{t,h}^{BLU}] + \tilde{\delta}_4^{h,3}[g_{t,h}^{GB} - g_{t,h}^{DRI}] + \tilde{\upsilon}_{t,h}^{GB}$

h	$\tilde{\delta}_0^{h,GB}$	$\tilde{\delta}_1^{h,GB}$	$\tilde{\delta}_2^{h,1}$	$\tilde{\delta}_4^{h,3}$	R^2	N
0	1.20 (.34)	.86 (.10)	.61* (.24)	- .43 (.28)	.63	127
1	.83 (1.04)	.96 (.34)	- .23 (.55)	- .62* (.38)	.23	127
2	2.32 (1.66)	.39 (.57)	.55 (.58)	- .26 (.41)	.07	127
3	3.02 (1.78)	.10 (.65)	.20 (.74)	- .10 (.30)	.02	127
4	- .22 (1.38)	1.42 (.49)	.97* (.46)	- .15 (.30)	.33	127
5	- .12 (1.32)	1.30 (.47)	1.07* (.70)	- .51 (.35)	.24	90
6	- 1.72 (2.06)	1.81 (.79)	- .02 (.44)	.35 (.24)	.42	53

3.2 *Other Publicly Known Information is Correlated with the Fed's Forecast Error*

We now explore other information which was known to the staff of the Federal Reserve and should have no impact on parameter estimates of (2a)-(2b). The main issue on which we focus is the monetary policy regimes in place at the time when the forecasts were made. To accomplish this we have defined two sets of variables which aim to measure the monetary regime in place:

(i) *Time Period*. We have divided the sample period into three sub-periods. Subperiod 1 is the pre-Volker period up to 1979:8, subperiod 2 consists of the Volker years 1979:9-1987:8 and subperiod 3 is the Greenspan era since 1987:9. Hence we define

$$1_j(t) = \begin{cases} 1 & \text{if} \quad t \in \text{sub-period } j \\ 0 & \text{otherwise.} \end{cases}$$

We were able to include the third monetary policy era in the analysis due to our extension of the original R&R (2000) data file from 1991:11 to 1995:11, giving us the very useful additional data. (ii) *Measures of monetary policy*. We utilize two measures of monetary policy. The first is the Federal Funds Rate *at the end of the previous month*, denoted by FF_{t-1}. To define the second variable we first identified the usual monetary regimes: the regime of *tightening monetary policy* defined by rising fund rates and the regime of *loosening monetary policy* defined by falling fund rates. To avoid ambiguity in transitions, a regime change is declared only after the Federal Funds rate changes by at least 25 basis points. To define CFF_{t-1} we first identify the monetary regime in place at date t-1. Then, CFF_{t-1} is the cumulative change of the Federal Funds rate from the start of this monetary regime until the end of date t-1 which is the end of the month prior to the date of forecast (hence t-1). Note that in all models we use monetary policy variables which are dated t-1 to ensure that regardless of the date of the month when the staff at the Fed or private forecasters make their forecasts, they know the exact state of the monetary variables defined here.

The variable FF_{t-1} tests if forecasters assessed correctly the effects of the Federal Funds rate on subsequent inflation and growth rates. To see what CFF_{t-1} measures, note that a 6% Federal Funds rate after a long period of rising interest rates (i.e., $CFF_{t-1} > 0$ and large) may be different from a 6% Federal Funds rate after a short period of rising rates. The long run average of CFF_{t-1} is zero, it moves away from zero as the duration of a regime increases and the largest values it takes occurs just before the turning points of regimes. Hence, a positive coefficient of CFF_{t-1} means that a cumulative decline of rates generate a *negative* forecast error and a cumulative rise of rates generate a *positive* forecast error. A positive forecast error says that, on average, forecasted inflation level is lower than the conditional mean realization. Hence a positive coefficient of CFF_{t-1} means the forecaster *overestimated*[6] *the cumulative effects of monetary policy on reducing inflation*: his forecasts of inflation were too high in response to cumulative falls of rates and too low in response to cumulative increases in rates.

[6] The terminology we adopt here may be confusing a bit. It is based on the fact that a high federal funds rate aims to *reduce* inflation. The fact is that a positive coefficient of either FF_{t-1} or CFF_{t-1} means that a positive value of FF_{t-1} or CFF_{t-1} leads to expected values which are *smaller* than conditional mean realizations and we would often think of this as an *underestimate* by a forecaster. Here we say that a forecaster overestimated the ability of funds rate or the cumulative funds rate to lower inflation since the aim of policy is to reduce the inflation rate and if the expected rate is lower, on average, than the realized rates we must interpret this to mean the forecaster overestimated the power of policy to attain its goal.

In addition to monetary policy variables, we also include as an explanatory variable, the forecast itself. This variable measures the degree to which a forecaster *incorrectly* assessed the impact of all other, unspecified, variables on future rates of inflation and GNP growth.

Given the above, the regression models which we test are as follows:

(4a) $$\pi_{t,h} = \sum_{j=1}^{3} 1_j(t)\left[\hat{\alpha}_{0,j}^{h,GB} + \hat{\alpha}_{1,j}^{h,GB}\pi_{t,h}^{GB} + \hat{\alpha}_{2,j}^{h,GB}FF_{t-1} + \hat{\alpha}_{3,j}^{h,GB}CFF_{t-1}\right] + \hat{\varepsilon}_{t,h}^{GB}$$

(4b) $$g_{t,h} = \sum_{j=1}^{3} 1_j(t)\left[\hat{\delta}_{0,j}^{h,GB} + \hat{\delta}_{1,j}^{h,GB}g_{t,h}^{GB} + \hat{\delta}_{2,j}^{h,GB}FF_{t-1} + \hat{\delta}_{3,j}^{h,GB}CFF_{t-1}\right] + \hat{\vartheta}_{t,h}^{GB}.$$

Sufficient number of observations are available for h = 0, 1, ..., 6 and for each horizon we estimate 12 parameters of each equation in (4a)-(4b). The study of these estimates is the central object of Section 5, where we evaluate the effects of monetary policy on forecast errors. Here it suffices to observe that since the variables considered in (4a)-(4b) were known to the Fed's staff, the null hypothesis is $\hat{\alpha}_{n,j}^{h,GB} = \hat{\delta}_{n,j}^{h,GB} = 0$ for n = 0, 2, 3 and $\hat{\alpha}_{1,j}^{h,GB} = \hat{\delta}_{1,j}^{h,GB} = 1$ for all j and all h. The results presented in Table 7A, Section 5, show the null hypothesis is strongly rejected by the data. Here, in Table 4, we only compare the adjusted correlation coefficients estimated for models (4a) - (4b) with corresponding coefficients, estimated for models (2a) - (2b). The table shows the forecasts

Table 4: Comparing Adjusted Correlation Coefficients for Fed's Staff Forecasts

	Inflation		*GNP Growth*	
Horizon h	Model (4a)	Model (2a)	Model (4b)	Model (2b)
0	.87	.83	.54	.53
1	.84	.72	.32	.25
2	.77	.60	.23	.18
3	.77	.54	.17	.08
4	.85	.54	.25	.16
5	.91	.54	.22	.18
6	.88	.59	.26	.22

and the publicly known monetary policy variables explain a statistically significant proportion of the Fed's staff forecast error and substantially improve the accuracy of the Fed's forecasts in Equations (2a)-(2b). This violation of the orthogonality conditions contradict the assumptions that the staff at the Fed utilized "the correct" forecasting model of the economy.

The results reported in Section 3.1 and those reported here are the same. In 3.1 we show that the Fed's forecast errors are correlated with private forecasts, known to the Fed at the time when the Green Book forecasts were prepared. The results in this Section show that the Fed's forecast errors are correlated with monetary policy information showing the Fed's staff *assessed incorrectly the impact of monetary policy on future inflation and GNP growth*. In either case the forecast errors of the Fed are correlated with information which was known when the forecasts were made. More generally, the staff used the "wrong" model. Recall that standard orthogonality conditions of rational expectations, assumed to be satisfied by all forecasters, are at the heart of the R&R (2000) argument in support of Fed's superior information. We have seen that the Fed's forecasts violated these orthogonality conditions and we show in Section 5 that neither the Fed nor private forecasters know the "true" forecasting function induced by the equilibrium. But then, the R&R (2000) conclusion of superior Fed information is flawed.

4. On the Heterogeneity of Forecasting Models

We turn now to the exploration of the evidence that the dominant reason for the diversity of forecasts is not private information but rather the diversity of forecasting models.

4.1 Illustrations of Forecast Heterogeneity

In Table 5A we present forecasts of GNP growth and inflation for 1991 made by participants in the Blue Chip Economic Indicators survey at the start of that year. It is important to note the quality of the forecasters. They consist of major U.S. corporations, financial institutions and consulting firms. Major production, investment and financial decisions are influenced by these forecasts and one must presume that these facts provide strong incentive for efficient use of information on part of the forecasters. It is thus rather extraordinary that about half of the private forecasters predicted in January 1991 that 1991 will be a recession year and the other half disagreed. The actual growth rate in 1991 was -.5% and the inflation rate 3.6%. Now, suppose you place yourself in January of 1991 and make a stationary econometric forecast of GNP growth without a non-stationary judgment about the conditions prevailing in 1991. We have done so using a model of Stock and Watson (1999a),(1999b),(2001) and estimating it by employing a

Table 5A: Blue Chip Forecasts of GNP Growth and Inflation for 1991

Forecasted Percent Change **January 1991** **Forecast for 1991**	**Real GNP**	**GNP Price Deflator**
Sears Roebuck & Co.	1.6H	4.2
Amhold & S. Bleichroeder	1.2	4.8
Prudential Bache	1.2	3.3L
Chicago Corporation	1.1	4.1
Bostian Economic Research	1.0	4.0
Faimodel	1.0	3.7
Cahners Economics	0.9	4.3
Wayne Hummer & Co. - Chicago	0.8	4.3
Nat'l. City Bank of Cleveland	0.7	4.6
Inforum - Univ. of Maryland	0.7	3.8
CRT Government Securities	0.6	4.0
Dun & Bradstreet	0.6	4.0
Conference Board	0.5	4.7
Econoclast	0.5	4.0
First National Bank of Chicago	0.5	3.8
Univ. of Michigan M.Q.E.M.	0.4	4.7
Manufacturers Natl. Bank - Detroit	0.3	4.5
Turning Points (Micrometrics)	0.2	4.3
Brown Brothers Harriman	0.2	4.0
Dean Witter Reynolds, Inc.	0.1	4.0
LaSalle National Bank	0.1	3.6
Northern Trust Company	0.0	4.3
Evans Economics	0.0	4.0
Morris Cohen & Associates	-0.1	5.0H
Prudential Insurance Co.	-0.1	4.5
Chrysler Corporation	-0.1	4.1
Econoviews International Inc.	-0.1	3.9
U.S. Trust Co.	-0.2	4.3
Reeder Associates (Charles)	-0.3	4.9
Siff, Oakley, Marks, Inc.	-0.3	4.8
Morgan Stanley & Co.	-0.3	4.7
Eggert Economic Enterprises, Inc.	-0.3	3.9
CoreStates Financial Corp.	-0.4	4.3
Mortgage Bankers Assn. of America	-0.4	4.3
Bank of America	-0.4	3.6
E.I. Du Pont de Nemours & Co.	-0.5	4.8
National Assn. of Home Builders	-0.5	4.5
Metropolitan Life Insurance Co.	-0.5	4.5
Ford Motor Company	-0.6	4.6
Chase Manhattan Bank	-0.6	4.0
U.S. Chamber of Commerce	-0.7	5.0H
Manufacturers Hanover Trust Co.	-0.7	4.4
Bankers Trust Co.	-0.7	4.4
Laurence H. Meyer & Assoc.	-0.7	4.0
Security Pacific Nat'l. Bank	-0.7	4.0
PNC Financial Corp.	-0.9	4.3
UCLA Business Forecast	-0.9	4.2
Merrill Lynch	-1.1	4.4
Georgia State University	-1.1	3.6
Equitable Life Assurance	-1.2	4.7
Morgan Guaranty Trust Co.	-1.2	3.8
Shawmut National Corp.	-1.3L	4.0

combination of diffusion indexes and averaged bivariate VAR forecasts utilizing a large number of U.S. time series. All non-judgmental stationary forecasts of GNP growth turned out *higher than most of the private forecasts*.

We shall argue later that in thinking about heterogeneity of forecasting models, it is useful to consider the non-judgmental stationary forecast as a yardstick, relative to which one measures the dynamic pattern of any rational forecaster. Rational deviations from the stationary forecasts indicates that an agent considers the circumstances, at the given date, as being sufficiently unique and different from the past that some adjustment is called for and hence the non-judgmental stationary forecast is not employed.

Picking another date, Table 5B exhibits the same information (but for GDP, rather than GNP) for May 2000. The actual growth rate in 2000 was 4.1% and the inflation rate 2.3%. It is

Table 5B: Blue Chip Forecasts of GNP Growth and Inflation for 2000

Forecasted Percent Change **May 2000** **Forecast for 2000**	**Real GDP**	**GDP Price Deflator**
First Union Corp.	5.3H	2.0
Turning Points (Micrometrics)	5.2	2.1
J P Morgan	5.2	2.1
Evans, Carroll & Assoc.	5.1	2.2
Mortgage Bankers Assn. of Amer.	5.1	2.1
Goldman Sachs & Co.	5.1	2.1
U.S. Trust Co.	5.1	2.0
US Chamber of Commerce	5.1	2.0
Banc of America Corp.	5.1	2.0
Morgan Stanley Dean Witter	5.1	1.9
Wayne Hummer Investments LLC	5.0	2.3
Bank One	5.0	2.1
Nomura Securities	5.0	1.9
Merrill Lynch	5.0	1.9
Perna Associates	4.9	2.3
National Assn. of Home Builders	4.9	2.1
Macroeconomic Advisers, LLC	4.9	2.1
Prudential Securities, Inc.	4.9	2.0
LaSalle National Bank	4.8	2.3
Conference Board	4.8	2.3
Wells Capital Management	4.8	2.2
DuPont	4.8	2.1
Northern Trust Company	4.8	2.1
Chicago Capital, Inc.	4.8	2.0
Deutsche Bank Securities	4.8	1.8
Chase Securities, Inc.	4.8	1.8
Credit Suisse First Boston	4.8	1.8
Comerica	4.7	2.4
Moody's Investors Service	4.7	2.2
Fannie Mae	4.7	2.0
Federal Express Corp.	4.7	2.0
SOM Economics, Inc.	4.7	1.9
National Assn. of Realtors	4.7	1.9
National City Corporation	4.7	1.9
Clear View Economics	4.7	1.9
Eggert Economic Enterprises, Inc.	4.6	2.1
WEFA Group	4.6	1.9
Eaton Corporation	4.6	1.9
Bear Stearns & Co., Inc.	4.6	1.2 L
Ford Motor Company	4.5	1.8
Motorola	4.5	1.7
Standard & Poors Corp.	4.5	1.7
UCLA Business Forecasting Proj.	4.4	2.1
Inforum - Univ. of Maryland	4.4	2.0
Prudential Insurance Co.	4.4	1.9
Weyerhaeuser Company	4.3	2.2
DaimlerChrysler AG	4.3	2.0
Georgia State University	4.2	2.2
Kellner Economic Advisers	4.2	2.0
Econoclast	4.1	2.0
Naroff Economic Advisors	4.0 L	2.5 H

thus surprising that in May of 2000, *five months into the year*, large variability in forecasts across agents is still exhibited and almost all the GDP forecasts turned out to be wrong. We repeated our experiment with stationary forecasting and this time the non-judgmental stationary forecast of

GDP growth rate turned out *lower than most of the private forecasts*. It is thus important to note that the distribution of forecasts fluctuates over time in relation to the stationary forecasts. We have seen that in 1991 private forecasters were conservative, placing their distribution *below* the stationary forecast while in May of 2000 their judgment was aggressive, placing the distribution *above* the stationary forecast.

Finally, we present in Figure 1 a chart of the distribution of private forecasts of annual GDP growth rate between 1990 and 2001. To explain this chart note that the forecasts are made in each quarter for GDP growth rate over the full year *following* the year of the forecast. Hence, in each quarter of a year the four forecasts are *about the same year*. For example in March, June, September and December of 1994 we have individual forecasts for the full year 1995. Figure 1

FIGURE 1: DISTRIBUTION OF PRIVATE FORECASTS OF GDP GROWTH

exhibits the 5th percentile and the 95th percentile of the forecast distribution for each quarter in which the forecasts were made. The horizontal bars in Figure 1 show the realization of GDP growth a year later. For example, the bar in 1999 exhibits the annual growth rate realized in 2000 and forecasted in the four quarters of 1999.

We observe that the distribution of forecasts fluctuates over time and exhibits a few very important properties:

(i) high degree of *persistence*, with serial correlation of forecasts and with fluctuating variance. Thus we observe *stochastic volatility of the distribution of beliefs*;

(ii) the distribution fluctuates over time but shows *no sign of convergence*;

(iii) the fact that the distribution itself fluctuates shows that there is *high degree of correlation among individual forecasters* over time.

The Bank of America, Merrill Lynch and Goldman Sachs & Co. have access to the same information used in forecasting inflation or GNP growth. Yet, at any date their forecasts are substantially different and *their rankings within the distribution of forecasters fluctuate over time*. It follows that these agents use different models to make forecasts and in Section 6 we briefly review the theory of Rational Beliefs, which explains this heterogeneity. Here we merely observe that heterogeneity of forecasting models is the norm in the market place and the distribution of forecasts shows no sign of convergence. But if market participants with the same information use different forecasting models, most or all of them do not make the "true" rational expectations forecasts. Also, the median private forecaster is a random agent whose forecasting record over any time interval may be superior or inferior to the Fed's record . Since neither the median forecast nor the Fed's forecasts satisfy orthogonality, it is not clear how one can arrive at conclusions regarding the information superiority of the Fed from a comparison between the Fed forecast and the median private forecast.

Our final task in the examination of forecast heterogeneity is to get a clearer view of the pattern of Fed forecasts in relation to the private distribution. We show that at any date there is a set of private forecasters who agree with the forecasts of the Fed.

4.2 At Any Date there is a Set of Private Forecasters Who Agree with the Fed's Forecasts

We now examine the location of the Fed's forecasts relative to the distribution of private forecasts of SPF and BLU. As explained earlier, we have quarterly forecast distributions for SPF and annual forecast distributions for SPF and BLU. The number of SPF forecasters fluctuates over time. Since the annual data combines SPF and BLU forecasters, the number of observations is typically between 60 and 100 and hence the annual data are more reliable.

For each forecasting horizon h and for each date t for which data is available, we compute the mean $\mu_{h,t}$ and the standard deviation $\sigma_{h,t}$ of the private forecasts. In Figure 2 we draw two bands: the upper band is $\mu_{h,t} + 2\sigma_{h,t}$ and the lower band is $\mu_{h,t} - 2\sigma_{h,t}$. Observations marked by • are the Fed's forecasts at that date. Our computations were carried out for all forecasting horizons and for full "next year" forecasts. The results for different forecasting horizons are very similar

FIGURE 2: FED'S FORECASTS RELATIVE TO THE DISTRIBUTION OF PRIVATE FORECASTS

and to conserve space we report here only three horizons: 1 and 3 future quarters and the full "next year" forecasts Hence, Figure 2 consists of two sets of charts: Figures 2.1- 2.3 are for inflation forecasting and Figures 2.4 - 2.6 are for GNP growth.

Inspection of Figure 2 reveals that the vast majority of the Fed's forecasts are distributed well within the $\mu_{h,t} \pm 2\sigma_{h,t}$ band of private forecasts. This result is particularly significant for the "next year" horizon for which we combine the SPF and BLU data so that the number of observations at each date is typically larger than 60 observations. With such high density of forecasts scattered within the band delineated in the charts it follows that at any moment of time there is a whole set of private forecasters who agree with the Fed forecasts.

Under the assumption of rational expectations the R&R (2000) methodology is applicable to *any* sequence of private forecasters, not only to the forecasts selected randomly by the criterion of being the median forecasts. Since at each date we have a group of commercial forecasters who agree with the Fed's forecasts, we can select for study, at each date, the forecasts of any member of this group. We can then estimate equations (1b) by using this sequence of forecast data instead of the data generated by the median forecaster. In that case the contribution of the Fed's forecasts would be zero and we would conclude that the Fed has no superior information. Indeed, if we found private forecasters with forecasting records over some time interval which is superior to the Fed's record, are we to conclude that over that time interval they had some private information about forecasting inflation or GNP growth which the Fed did not have?

4.3 Fed's Forecasts Fluctuate Less than Private Forecasts.

Suppose the diversity of forecasts arises from the fact that agents believe the economy is a non-stationary process in which the structure changes over time. In that case rational agents may have diverse, time dependent, forecast functions with conditional forecasts which fluctuate over time. Moreover, the cross sectional distribution of private forecasts would be time dependent and fluctuate as in Figure 1.

Suppose, however, that we normalize the distribution at each date to have zero mean and unit variance. Suppose also that the Fed's forecast function *generates forecasts which are independent draws from the distribution of private forecasts*. It then follows that the frequency distribution of normalized Fed forecasts *over time* should be the same as the distribution of private forecasts. More specifically, we defined the two variables

$$Z^{\pi}_{h,t} = \frac{\pi^{GB}_{h,t} - \mu^{\pi}_{h,t}}{\sigma^{\pi}_{h,t}} \qquad , \qquad Z^{g}_{h,t} = \frac{g^{GB}_{h,t} - \mu^{g}_{h,t}}{\sigma^{g}_{h,t}}.$$

$(\mu^{\pi}_{h,t}\ ,\ \mu^{g}_{h,t})$ are sample means of inflation and GNP growth and $(\sigma^{\pi}_{h,t}\ ,\ \sigma^{g}_{h,t})$ are the corresponding standard deviations of the distributions of *private forecasts* for horizon h at date t. Under the above hypothesis $(Z^{\pi}_{h,t}\ ,\ Z^{g}_{h,t})$ are approximately normally distributed with *zero mean and unit variance*. We would thus expect that for each h the moments of $(Z^{\pi}_{h,t}, Z^{g}_{h,t})$ over the time series should satisfy $\overline{Z}^{\pi}_{h} = 0\ ,\ \overline{Z}^{g}_{h} = 0$ and $\sigma_{Z^{\pi}_{h}} = 1\ ,\ \sigma_{Z^{g}_{h}} = 1$. These sample statistics reveal the location of the Fed forecasts within the distribution of private forecasts. We computed these statistics for h = 1 , h = 3 and for "next year" forecasts. We also computed them for two sub-periods: the pre-Volker era 1968:11-1979:8 and for 1979:9-1995:11. Distributional data for GNP growth and full year forecasts are not available for the first period. We now report the results.

<u>1968:11-1995:11</u>

	Inflation	
	Mean	Standard Deviation
h = 1	-.05	.39
h = 3	-.23	.35

<u>1968:11- 1979:8</u>

	Inflation	
	Mean	Standard Deviation
h = 1	.15	.25
h = 3	.04	.16

<u>1979:09- 1995:11</u>

	Inflation		GNP Growth	
	Mean	Standard Deviation	Mean	Standard Deviation
h = 1	-.22	.45	-.02	.44
h = 3	-.43	.40	-.26	.50
Next year	-.72	.58	-.25	.50

The results show that for all estimates $\sigma_{Z^{\pi}_{h}} < 1\ ,\ \sigma_{Z^{g}_{h}} < 1$. The means are close to zero for short horizon but fluctuate over time: in some periods they are positive and in others they are negative. The only significant pattern can be noted during the post Volker era when Fed forecasts were consistently on the low side of the private distribution.

We conclude that the variance of the Fed forecasts is *much* smaller than can be predicted by the cross sectional variance of private forecasters. That is, the Fed's staff forecasts *tended to be more conservative, with a smaller time variability than private forecasts*. This observation will become more important later, when we discuss the "rationality" conditions mentioned earlier.

We qualify this conclusion in two ways. First, the number of quarterly Fed forecasts available at the same dates when an SPF forecast distribution is available, is not large. For

1968:11 - 1979:8 we have 42 observations for h = 1 and 36 observations for h = 3 while for 1979:9 - 1995:11 we have 49 observations for h = 1, 3. The situation is better with respect to the full “next year” forecast, where we have 108 Fed forecasts since then we combined the SPF with the BLU “next year” forecasts. Second, we have no evidence that private forecasts come from a fixed distribution as postulated above hence the variances used in the computations of $(Z^{\pi}_{h,t}, Z^{g}_{h,t})$ may not represent any particular distribution. Indeed, we have already seen that the distributions of private forecasts fluctuate over time. Time variations in the mean forecast is to be expected, but we also observe stochastic volatility in the form of time variability of the variances. We thus view the results of this sub-section as very preliminary.

In conclusion, we have exhibited in this section a great diversity of private forecasts. Given that the Fed’s forecast functions do not satisfy orthogonality, the evidence is that the Fed used its own subjective model to forecast inflation and GNP growth. No economic agent, including the Fed, knows the “true” forecast function: heterogeneity of forecasting models and beliefs is the norm in the market-place.

5. Tests of Inflation Forecast Error Orthogonality

In this section we study in detail the structure of inflation forecast errors of the Fed and of private forecasters in relation to monetary policy variables and other information implicit in the forecasts themselves. This entails an examination of the forecast errors in equations like (4a) for the Fed and for private forecasters. Before formulating the models to be estimated, we motivate this examination.

5.1 The Puzzling Question of “Rationality” Conditions $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ *and* $\alpha_1^{h,k} = \delta_1^{h,k} = 1$

The conclusion of belief heterogeneity in the market place leads to a puzzling observation. On the one hand we have seen in Sections 3 - 4 that the Fed and private forecasters utilize diverse forecasting models *all of which violate orthogonality*. On the other hand we noted in Section 3.1 (see discussion following Table 2) that *over the long term*, the Fed’s forecasting is compatible with the hypothesis $\alpha_0^{h,GB} = \delta_0^{h,GB} = 0$ and $\alpha_1^{h,GB} = \delta_1^{h,GB} = 1$ for all h. We noted that this question is examined by estimating (2a)-(2b) for the entire sample. For the Fed forecasts we reported these results in Table 2. We now report in Table 6 analogous results for private forecasters.[7]

The results in Table 6 show the data are compatible with the hypothesis $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ and $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ *only for very short horizons*: 3 quarters or less into the future for inflation[8] and less than 2 quarter into the future for GNP growth. The divergence of the estimates from the values $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ and $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ *increases* with h. These facts raise a simple question. If

Table 6: Accuracy of Private Forecasts, Estimates of (2a)-(2b)

(standard errors in parentheses)

	INFLATION				GNP GROWTH			
	α_0	α_1	R^2	N	δ_0	δ_1	R^2	N
BLU								
h = 0	-.40 (.18)	.99 (.05)	.89	252	.78 (.45)	1.08 (.14)	.42	252
h = 1	-.48 (.26)	.96 (.07)	.84	249	.68 (.86)	1.01 (.29)	.20	249
h = 2	-.49 (.35)	.92 (.10)	.77	246	2.64 (1.08)	.25 (.42)	.00	246
h = 3	-.37 (.40)	.86 (.11)	.71	243	4.01 (1.44)	- .23 (.56)	.00	243
h = 4	.29 (.30)	.64 (.08)	.61	235	3.23 (1.39)	.03 (.54)	.00	235
h = 5	.23 (.27)	.60 (.07)	.63	174	2.99 (1.63)	.14 (.63)	.63	174
h = 6	.45 (.23)	.50 (.06)	.61	114	1.81 (1.79)	.56 (.68)	.00	114
h = 7	.78 (.19)	.41 (.03)	.72	57	1.06 (2.82)	1.00 (1.02)	.02	57
SPF								
h = 0	-.18 (.22)	1.05 (.05)	.83	129	.35 (.42)	1.25 (.14)	.52	78
h = 1	-.14 (.29)	1.05 (.07)	.69	128	-.59 (.95)	1.48 (.32)	.27	77
h = 2	-.18 (.40)	1.06 (.10)	.57	127	1.78 (1.12)	.60 (.43)	.03	76
h = 3	.23 (.52)	.96 (.11)	.43	126	2.62 (1.41)	.32 (.53)	.00	75
h = 4	.55 (.55)	.89 (.13)	.37	120	2.52 (1.41)	.38 (.54)	.00	74
DRI								
h = 0	.35 (.20)	.93 (.04)	.80	248	.98 (.25)	.93 (.07)	.56	272
h = 1	.65 (.32)	.90 (.07)	.66	248	1.07 (.49)	.78 (.13)	.27	272
h = 2	.44 (.36)	.93 (.08)	.59	248	1.29 (.49)	.66 (.14)	.14	272
h = 3	.82 (.46)	.82 (.10)	.49	248	1.65 (.60)	.47 (.18)	.05	272
h = 4	1 .29 (.56)	.71 (.12)	.34	248	1.78 (.57)	.35 (.18)	.03	272
h = 5	1 .67 (.62)	.63 (.13)	.25	248	1.88 (.66)	.63 (.19)	.02	272
h = 6	2 .18 (.75)	.51 (.14)	.15	248	3.61 (.76)	-.12 (.23)	.00	272
h = 7	2 .55 (.80)	.41 (.15)	.09	246	4.91 (.85)	-.51 (.26)	.03	27

[7] R&R (2000) report similar results (see their Table 1, page 434) for the inflation forecast data of private forecasters but only for the shorter period which ends in 1991:11.

[8] Since we use *second revision* data the Keane and Runkle (1990) argument against Zarnowitz (1985), regarding the need to use first release data, does not appear to apply to the forecasting data of the median SPF forecaster. The results in Table 6 show that for the very short horizons the conditions $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ and $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ are satisfied, in accord with the results of Keane and Runkle (1990) who used *first release* data. However, the fact is that satisfaction of these "rationality" conditions does not imply Rational Expectations.

violation of orthogonality for all horizons implies a rejection of rational expectations, why is it that for some forecasters and for some horizons, the conditions $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ and $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ are satisfied? Also, it is puzzling why these conditions are satisfied for private forecasters only *for short forecasting horizons* but are violated for long forecasting horizons. We return to these questions after we study the pattern of forecast errors of the Fed and of private forecasters.

5.2 Estimating the Effect of Monetary Policy Variables on Inflation Forecast Errors.

We now reformulate model (4a)-(4b) and study the following equations for the Fed's staff and for private forecasters

$$\text{(5a)} \qquad \pi_{t,h} - \pi_{t,h}^k = \sum_{j=1}^{3} 1_j(t)\left[\hat{\alpha}_{0,j}^{h,k} + \hat{\alpha}_{1,j}^{h,k}\pi_{t,h}^k + \hat{\alpha}_{2,j}^{h,k}FF_{t-1} + \hat{\alpha}_{3,j}^{h,k}CFF_{t-1}\right] + \hat{\varepsilon}_{t,h}^k$$

$$\text{(5b)} \qquad g_{t,h} - g_{t,h}^k = \sum_{j=1}^{3} 1_j(t)\left[\hat{\delta}_{0,j}^{h,k} + \hat{\delta}_{1,j}^{h,k}g_{t,h}^k + \hat{\delta}_{2,j}^{h,k}FF_{t-1} + \hat{\delta}_{3,j}^{h,k}CFF_{t-1}\right] + \hat{\vartheta}_{t,h}^k .$$

In (5a)-(5b) we propose to "explain" the forecast errors by using (i) time variables covering the three periods under study, (ii) monetary policy variables (FF_{t-1}, CFF_{t-1}), known at t and (iii) the forecasts themselves. Hence, the null hypothesis in all tests is $\hat{\alpha}_{n,j}^{h,k} = \hat{\delta}_{n,j}^{h,k} = 0$ for all indexes.

Interpretation of Estimated Parameters

To explain how to interpret the results, recall that since we are evaluating forecast *errors*, a correct reading of the results in the tables below requires us to focus on what the forecaster *missed* or assessed *incorrectly*. To do that *we use time as an important reference*. For each forecaster we want to detect *systematic* patterns of forecasting errors that prevailed in each period, based on the model each forecaster used at that time. In a non-stationary world where technology, social institutions and political environment change, it is difficult to know what the "true" or "correct" forecasting function is. If an economic environment lasts long enough, one can assume that agents will adapt and learn how to make more accurate forecasts. However, if the environment keeps changing, they never converge and never attain perfect knowledge. In that case we could only expect a sequence of forecast errors which are different from time period to another, while the agents adapt and do the best they can in formulating their subjective models.

The forecasts $(\pi_{t,h}^k, g_{t,h}^k)$ are used as explanatory variables in order to estimate the failure of orthogonality with respect to *all other information* available at t, incorporated in $(\pi^{k}_{t,h}, g^{k}_{t,h})$ but not explicit in (5a)-(5b). The advantage of using the forecasts themselves is that the parameters $(\hat{\alpha}^{h,k}_{0,j}, \hat{\delta}^{h,k}_{0,j})$ and $(\hat{\alpha}^{h,k}_{1,j}, \hat{\delta}^{h,k}_{1,j})$ tell us something about the nature of the mistaken models used by the forecasters. The typical result for inflation is $\hat{\alpha}^{h,k}_{0,j} > 0$ and $\hat{\alpha}^{h,k}_{1,j} < 0$, as can be seen later, and we illustrate the interpretation of the parameters by explaining the meaning of this pattern.

(i) <u>A positive estimate</u> of $\hat{\alpha}^{h,k}_{0,j}$ means that during period j and for horizon h, forecaster k generated a mean forecast that was, *on average,* lower than the mean inflation realized later in the market. We then say that during period j and for horizon h the *intercept* of the forecast function was too low and thus the forecast *underestimated* the mean inflation.

(ii) <u>A negative estimate of</u> $\hat{\alpha}^{h,k}_{1,j}$ is interpreted to mean that during period j and for horizon h, forecaster k generated a forecast function with an upward biased slope. That is, for each 1% increment in forecasted inflation rate, the realized rate was on average smaller than 1%. We say that during period j and for horizon h the *slope* of the inflation forecast was upward biased: when the forecaster did forecast inflation, his forecast was systematically too high.

To combine the above two interpretations, $\hat{\alpha}^{h,k}_{0,j} > 0$ and $\hat{\alpha}^{h,k}_{1,j} < 0$ mean first that the forecaster underestimated the mean inflation by *failing to forecast it* thus forecasting lower average inflation. In addition, these say that when his model actually forecasted inflation, it tended to generate too high forecasts: for each forecasted rate the realized rate was lower.

We turn to the interpretation of the parameter of FF_{t-1}. It describes quantitatively how well forecaster k was able to assess the impact of changes in the federal funds rate, during period j, on future inflation rates over horizon h. Keep in mind that a rising funds rate is associated with a *decline* in future inflation. Hence, a *positive* estimate says that a high funds rate leads to a systematic positive forecast error, or to systematically larger realized inflation than forecasted. Thus the forecasted inflation was too low and this means an *overestimation* of the impact of high funds rate on reducing future inflation. Similarly, a *negative* parameter means that the forecaster did not assess correctly the effect of monetary policy and *underestimated* the ability of Fed to reduce inflation by raising rates.

Finally, we have discussed CFF_{t-1} earlier. Recall that a positive coefficient of CFF_{t-1} in an equation means a cumulative decline of rates generate a *negative* forecast error and a cumulative rise of rates generate a *positive* forecast error. Hence the forecaster *overestimated the cumulative effects of monetary policy on reducing inflation*: his forecasts were too inflationary in response to cumulative decline of rates and predicted too large a decline of inflation rates in response to cumulative increases in rates.

Evaluation of the Results

Since each equation has 12 parameters, given a forecasting horizon of up to 6 quarters there are up to 84 parameters for inflation forecasting and up to 84 parameters for GNP growth. The results for inflation are analogous to those for GNP growth but the estimates for GNP growth are less reliable. We thus study in the text only the results for the inflation forecast error estimates of (5a) while the results for (5b) are reported in an Appendix. Also, to conserve space and avoid repetition we provide estimates only for SPF and BLU; the conclusions for DRI are similar. The results for inflation are in Tables 7A -7C and for GNP growth, in Tables numbered 7D -7F in the Appendix. The symbol (*) means that the parameter is statistically significantly different from zero at least at the 10% confidence level. For each horizon from 0 through 6 the parameters are arranged in four groups. Reading down each column we report first the *three constants*, second the *three parameters of the forecasts*, third the *three parameters of the federal funds rate* and finally the *three parameters of the cumulative funds rate*. This way of reporting the parameters enables a comparison of the forecast errors across the three monetary regimes.

Commenting on all Tables combined, observe that the results are statistically significant for all forecasters. A large number of statistically significant parameters are found in each of the tables, indicating a significant correlation between the specified variables and the forecast errors of the Fed and of private forecasters. Indeed, for longer horizons *the variables used account for over 50% of all inflation forecast errors!!* The results for horizon of $h = 6$ are not reliable due to the relatively small number of observations, particularly in the first era. We now turn to the specific tables, studying in detail the results for the Fed's forecasts, reported in Table 7A.

The Fed's forecasts during the 1965-1979 monetary regime:

(i) the *mean* of the Fed's long horizon forecasts were downward biased: the average realized inflation was dramatically higher than the average Fed's forecasts;

(ii) the slopes of the Fed's forecasts were systematically biased upward: for every percentage point of forecasted inflation, realized inflation was *on average* proportionally lower. This follows from $\hat{\alpha}_{1,1}^{h,GB} < 0$. The proportional bias *increased* with the forecasting horizon;

(iii) the Fed *overestimated* the impact of higher interest rates on reducing inflation over short horizons. It is seen by the positive coefficient of the funds rate for short horizons: the mean decline in inflation in response to higher funds rate was, on average, not as large as the Fed expected;

Table 7A: Test of Fed's Inflation Forecast Error Orthogonality with Monetary Policy Variables

(standard errors in parentheses)

Horizon →	0	1	2	3	4	5	6
$\hat{\alpha}_{0,1}^{h,GB}$	-.03	-.02	1.46	3.01*	6.06*	11.11*	20.30*
	(.43)	(.70)	(1.14)	(1.17)	(1.18)	(1.19)	(1.30)
$\hat{\alpha}_{0,2}^{h,GB}$	-.02	.70*	.11	.28	.15	.47	1.21
	(.61)	(.41)	(.51)	(.48)	(.62)	(.58)	(1.02)
$\hat{\alpha}_{0,3}^{h,GB}$	.02	-.17	-.21	.28	.13	.36	.65
	(.30)	(.36)	(.35)	(.42)	(.38)	(.43)	(.43)
$\hat{\alpha}_{1,1}^{h,GB}$	-.29*	-.34*	-.41*	-.46*	-.79*	-1.45*	-.54*
	(.06)	(.11)	(.15)	(.14)	(.22)	(.21)	(.22)
$\hat{\alpha}_{1,2}^{h,GB}$	-.10	.05	.09	.09	.15	-.03	-.48*
	(.10)	(.06)	(.08)	(.08)	(.14)	(.14)	(.25)
$\hat{\alpha}_{1,3}^{h,k}$	-.39*	-.42*	-.03	-.03	.20	-.12	-.39*
	(.12)	(.14)	(.17)	(.17)	(.16)	(.24)	(.19)
$\hat{\alpha}_{2,1}^{h,GB}$	.27*	.42*	.32*	.13	-.04	- .10	-1.76*
	(.08)	(.14)	(.19)	(.19)	(.13)	(.07)	(.08)
$\hat{\alpha}_{2,2}^{h,GB}$	.02	-.14*	-.11*	-.14*	-.17*	-.14*	-.01
	(.08)	(.04)	(.04)	(.04)	(.07)	(.07)	(.10)
$\hat{\alpha}_{2,3}^{h,GB}$	.15*	.20*	-.01	-.08	-.17*	-.06	-.01
	(.07)	(.09)	(.10)	(.10)	(.10)	(.10)	(.11)
$\hat{\alpha}_{3,1}^{h,GB}$	-.04	-.03	.08	.27*	.49*	.60*	.78*
	(.05)	(.07)	(.10)	(.09)	(.10)	(.06)	(.05)
$\hat{\alpha}_{3,2}^{h,GB}$	-.01	.03	.02	.07*	.02	.05	.11*
	(.05)	(.02)	(.03)	(.04)	(.05)	(.04)	(.06)
$\hat{\alpha}_{3,3}^{h,GB}$	-.03	-.04	.01	.04	.03	.01	.01
	(.03)	(.04)	(.04)	(.04)	(.03)	(.03)	(.04)
R^2	.28	.42	.43	.51	.68	.81	.71
N	294	278	256	239	209	150	90

(iv) in accord with our interpretation of CFF_{t-1}, during the first era this coefficient is positive for long horizons and hence the Fed *overestimated* the cumulative effects of monetary policy on *long* *forecast horizons*. The Fed exhibited excessive inflationary forecasts in response to cumulative falls of rates and forecasted too low inflation in response to cumulative increases in rates.

In summary, during this era of rising commodity prices the Fed forecasters failed dramatically to forecast the mean inflation. However, when forecasting inflation their model had an upward biased slope. As for monetary policy, conclusion (iv) is compatible with (iii) and reinforces it, drawing a picture of the Fed *overestimatin*g the impact of tight monetary policy on inflation over all horizons.

The Fed's forecasts during the Volker monetary regime

(i) during the Volker era the Fed forecasters assessed correctly the mean inflation and the slope of their forecast function was correct;

(ii) in contrast with the earlier pattern during this era, the Fed *underestimated* the impact of higher interest rates on lowering inflation over all horizons. This is seen by the consistently negative coefficient of the funds rate;

(iii) since some long horizon coefficients of CFF_{t-1} are positive the Fed *overestimated* the cumulative effects of monetary policy on long horizon inflation: their inflationary forecasts were too high in response to cumulative decline of rates and too low in response to cumulative increases in rates. This effect is generally small.

In summary, during this era the Fed's inflation forecasting exhibited no bias in intercept or slope but they were consistently wrong is assessing the impact of monetary policy. We have shown the Fed forecasters *underestimated* the impact of tight monetary policy on inflation over all horizons. However, we also find limited *overestimation* of the cumulative effect of tight money on *long horizon* inflation rates. We reconcile these conclusions by noting that result (ii) reflects a broad underestimation of the impact of tight money policy on inflation. This may suggest to some that the willingness of the Fed at that time to engage in an aggressive tight money policy was bolstered by forecasts which underestimated the effect of such policy. Result (iii) reflects the fact that this underestimation bias tended to weaken after a *long period* of rising rates. The bias is somewhat decreased by the small contrary effect of CFF_{t-1}.

The Fed's forecasts during the Greenspan monetary regime

(i) the slopes of the Fed's very short horizon forecasts were biased upward, returning to the pattern of the 1965-1979 monetary era. For each 1% forecasted increase in the inflation rate over short horizons realized increased inflation was *on average* less than 1%;

(ii) the Fed *overestimated* the impact of higher interest rates on inflation only over short horizons reflected in $\hat{\alpha}_{1,3}^{h,k} < 0$ for h = 0, 1.

In summary, during this era the Fed forecasters consistently forecasted too high inflation rates over short horizons but showed no such bias over long horizons. They consistently overestimated the ability of higher interest rates to lower inflation over short horizons. We thus conclude the Fed forecasters incorrectly *expected policy to impact the economy earlier than it did.*

Turning now to the results in Tables 7B - 7C note first that since SPF forecasts cover only four future quarters, the forecasting horizon in Table 7B is shorter than in 7A. As for BLU, since the data available from this source starts in 1980, Table 7C covers only the second and third monetary regimes. With these differences clarified, observe that a comparison of Tables 7A, 7B and 7C reveals that the pattern of statistically significant coefficients is surprisingly similar in all three tables. Hence the forecasting errors of the Fed's staff and those of the private forecasters had similar characteristics. We briefly comment first on the median SPF forecaster.

During the 1965-1979 monetary regime:

(i) the *intercept* of the median SPF longer horizon forecasts (3 - 4 quarters) were downward biased reflected in $\hat{\alpha}_{0,1}^{h,SPF} > 0$ for longer horizons;

(ii) the median SPF forecasts had a positively biased slope reflected in $\hat{\alpha}_{1,1}^{h,SPF} < 0$ for all horizons. The bias *increased* with the forecasting horizon;

(iii) the median SPF *overestimated* effect of high interest rates on lower inflation over short horizons;

(iv) the median SPF *overestimated* the cumulative effects of monetary policy on *longer horizons*.

In sum, the qualitative pattern for the median SPF forecaster is essentially the same as for the Fed.

During the Volker monetary regime

(i) the median SPF *underestimated* the effect of higher interest rates on reducing inflation over all horizons;

(ii) the median SPF *overestimated* the cumulative effects of monetary policy on reducing inflation in longer forecasting horizons (3 - 4 quarters).

Here again, the qualitative pattern of the median SPF forecast error is essentially the same as the Fed's.

Table 7B: Test of Median SPF Inflation Forecast Error Orthogonality with Monetary Policy Variables

(standard errors in parentheses)

Horizon →	0	1	2	3	4
$\hat{\alpha}_{0,1}^{h,SPF}$	.64 (.86)	.33 (.85)	1.55 (1.20)	3.34* (1.22)	3.90* (1.36)
$\hat{\alpha}_{0,2}^{h,SPF}$	-.52 (.49)	-.17 (.60)	-.88 (.74)	-.70 (.90)	1.42* (.68)
$\hat{\alpha}_{0,3}^{h,SPF}$	-.39 (.41)	-.53 (.47)	-.85* (.50)	-.74 (.52)	-.65 (.54)
$\hat{\alpha}_{1,1}^{h,SPF}$	-.32* (.10)	-.47* (.13)	-.47* (.15)	-.59* (.14)	-.65* (.19)
$\hat{\alpha}_{1,2}^{h,SPF}$	-.01 (.17)	.27* (.14)	.38 (.25)	.21 (.20)	.18 (.24)
$\hat{\alpha}_{1,3}^{h,SPF}$	-.25* (.13)	-.29* (.16)	-.37* (.19)	-.35* (.20)	-.41* (.24)
$\hat{\alpha}_{2,1}^{h,SPF}$	.29* (.15)	.49* (.14)	.33 (.20)	.17 (.21)	.16 (.15)
$\hat{\alpha}_{2,2}^{h,SPF}$	.01 (.08)	-.20* (.08)	-.22* (.11)	-.18* (.11)	-.38* (.12)
$\hat{\alpha}_{2,3}^{h,SPF}$	.13 (.09)	.16* (.10)	.23* (.10)	.19* (.11)	.20* (.09)
$\hat{\alpha}_{3,1}^{h,SPF}$	-.01 (.08)	.02 (.08)	.13 (.11)	.26* (.10)	.43* (.10)
$\hat{\alpha}_{3,2}^{h,SPF}$	.04 (.05)	.09* (.05)	.00 (.07)	.22* (.07)	.20* (.10)
$\hat{\alpha}_{3,3}^{h,SPF}$	-.05 (.04)	-.05 (.04)	-.08* (.05)	-.05 (.05)	-.06 (.04)
R^2	.30	.54	.54	.58	.67
N	129	128	127	126	120

During the Greenspan monetary regime

(i) the median SPF forecasts were strongly biased upward for all forecasting horizons;

(ii) the median SPF *overestimated* the effect of higher interest rates on reducing inflation over all horizons. This is reflected in $\hat{\alpha}_{2,3}^{h,SPF} > 0$ for all horizons. Recall that for the Fed this systematic forecasting mistake is exhibited only for short horizons while for the median SPF it is present for all horizons. This is the main difference between the Fed's error and the median SPF error.

Finally we comment on the BLU forecast errors. An examination of Table 7C shows that restricting attention only to the two monetary regimes after 1980, the main *differences* between the pattern of forecast errors of the median BLU and those of the Fed are as follows:

(i) the *intercept* of the median BLU forecasts of inflation rates for moderate horizons are upward biased during the Greenspan era (i.e. $\hat{\alpha}_{0,3}^{h,BLU} < 0$) and hence during that era the mean inflation rate was lower than the average forecasts of the median BLU. This pattern does not hold for the Fed or for the median SPF forecasts;

(ii) the slopes of the median BLU forecasts were downward biased for medium horizons during the Volker era;

(iii) similar to the median SPF, the median BLU *overestimated* the effect of higher interest rates on reducing inflation over all horizons during the Greenspan era. This is the main difference between the Fed's error and the forecast errors of the private forecasters.

Table 7C: Test of Median BLU Inflation Forecast Error Orthogonality with Monetary Policy Variables

(standard errors in parentheses)

Horizon →	0	1	2	3	4	5	6
$\hat{\alpha}_{0,2}^{h,BLU}$	-.38	-.39	-.99*	-.96	-.24	.22	2.17*
	(.37)	(.46)	(.53)	(.65)	(.64)	(.79)	(1.04)
$\hat{\alpha}_{0,3}^{h,BLU}$	-.38	-.56	-.77*	-.76*	-.70	-.73	-.46
	(.37)	(.40)	(.41)	(.45)	(.49)	(.48)	(.51)
$\hat{\alpha}_{1,2}^{h,BLU}$	.12	.50*	.68*	.60*	.16	-.06	-1.07*
	(.10)	(.14)	(.18)	(.20)	(.20)	(.25)	(.38)
$\hat{\alpha}_{1,3}^{h,k}$	-.23*	-.28*	-.30*	-.24	-.32*	-.31*	-.23
	(.13)	(.15)	(.17)	(.19)	(.20)	(.19)	(.17)
$\hat{\alpha}_{2,2}^{h,BLU}$	-.08	-.32*	-.39*	-.38*	-.25*	-.19*	.16
	(.06)	(.09)	(.08)	(.09)	(.10)	(.10)	(.15)
$\hat{\alpha}_{2,3}^{h,GB}$	.11	.15*	.18*	.15*	.16*	.12*	.01
	(.09)	(.09)	(.09)	(.09)	(.08)	(.07)	(.08)
$\hat{\alpha}_{3,2}^{h,BLU}$	.05	.05*	.09*	.16*	.00	-.00	.01
	(.04)	(.03)	(.04)	(.05)	(.05)	(.06)	(.05)
$\hat{\alpha}_{3,3}^{h,BLU}$	-.04	-.05	-.05	-.03	-.04	-.01	.01
	(.03)	(.04)	(.04)	(.04)	(.04)	(.03)	(.03)
R^2	.05	.26	.36	.38	.44	.52	.65
N	252	249	246	243	235	174	114

We have thus arrived at a surprising conclusion. Although there is strong evidence for great heterogeneity among beliefs and forecasting models of agents, we also find substantial similarity in the *qualitative* patterns of forecast errors of the three market participants considered here. Qualitative similarity of forecast errors does not mean they make the same *quantitative* forecasts, but it does imply similarity in the basic concepts and ideas underlying the forecasting models of market participants *even when these ideas are wrong*. This implies a degree of correlation among the beliefs of divergent agents in the economy confirming earlier observations made in connection with Figure 1. Correlation *among forecast errors* of agents in the economy is a powerful force generating market volatility which Kurz (1974),(1997) calls "Endogenous Uncertainty" (for more details see Kurz and Motolese (2001)).

In comparison with the qualitative similarity of the forecast errors of agents, the noted differences are not large. The main difference between the Fed's and the private forecast errors is the assessment of the effect of high interest rates on lowering inflation rates. While private forecasters *underestimated* the effect of the Volker high interest rate regime, they have turned around and *overestimated* the effect of monetary policy during the Greenspan era, *expecting policy under Greenspan to be more effective than it actually was.*

6. Explaining the Pattern of the "Rationality" Conditions

The evidence reveals the presence of extensive diversity forecasting models which violate orthogonality. How do we reconcile model diversity for which forecast errors are correlated with known information, with the fact that on average, the conditions $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ and $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ are satisfied by the Fed forecasters for all h and for private forecasters for short horizon values of h? These findings clearly violate rational expectations but we observe that satisfaction of the "rationality" conditions $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ and $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ does not imply that agents hold rational expectations! In the rest of this section we argue that both the observed diversity of beliefs as well as the empirical facts outlined here are compatible with the predictions of the theory of Rational Beliefs (in short RB; see Kurz (1994), (1997)). Hence our conclusions support the hypothesis that agents hold RB. We briefly explain the basic ideas of the RB theory.

6.1 Rational Beliefs

A detailed account of the RB theory may be found in Kurz (1997) or Kurz and Motolese (2001). In addition, we provide in Appendix B a short formal statement the definition of RB with some comments which may assist the reader. Here we briefly explain that the RB theory was developed to account for situations where an economic environment is non-stationary due to changes in technology, economic institutions, political environment and changes in economic policy regimes. The RB theory *assumes* that agents do not know the true probability underlying the equilibrium process but they have a great deal of past data about the observable variables in the economy. Using past data agents compute the empirical distribution and construct from it a probability measure over infinite sequences of observable variables. It can be shown that this probability is stationary and we call it "the stationary measure." Since all agents can carry out such computations, the stationary measure is known to all and consequently agents *do not disagree about the stationary forecasts*. All agents who believe the economic environment is stationary adopt the empirical distribution (or the implied stationary measure) as their belief.

Since the true non-stationary equilibrium process is not known, agents who believe the dynamics of the economy is non-stationary construct their own subjective probability models. A *Rational Belief* is then a probability over infinite sequences of observables, perhaps non-stationary, which *cannot be contradicted by the empirical evidence*. If one uses an RB model to simulate the data of the economy it will reproduce the known empirical distribution of equilibrium observables. An RB may imply forecasts which are, at times, higher or lower than the stationary forecasts but it is irrational to forecast *at all dates* above or *at all dates* below the stationary forecasts. Rationality of beliefs thus requires that given any observed state, conditional forecasts are empirically correct, on average. More precisely, they are compatible with stationary forecasts in the sense that the time average of the RB conditional forecasts must be equal to the conditional stationary forecasts. This is also true for all unconditional forecasts. It can thus be shown that agents who hold RB and do not believe the environment is stationary, generate forecasts which fluctuate around the stationary forecasts. Hence, an economy in which agents hold heterogenous RB must exhibit excess fluctuations due to fluctuations in the states of beliefs of the agents. This leads to an important principle which holds that *diversity of correlated* *rational beliefs implies volatility*!

The above discussion shows that non-stationary RB act as in the data: conditional upon the observables, forecasts fluctuate over time around the stationary forecast without a tendency for convergence. The RB theory says that the forecasts in Tables 5 - 6 and Figures 1- 2 are made by agents who believe the dynamics of the economy is non-stationary. The empirical evidence has revealed they use correlated subjective heterogenous models, combining past data with subjective assessment of the unique conditions prevailing at each date. But then, what about orthogonality?

Orthogonality Conditions Under Rational Beliefs

The basis for orthogonality under rational expectations is the fact that the conditional forecasts of a future random variable z_{t+h} under a true probability Π is orthogonal to any date t observable variable x_t contained in the information I_t, in the sense that at date t we can predict

(6a) $$E^{\Pi}[(z_{t+h} - E^{\Pi}(z_{t+h}|I_t))(x_t - \bar{x})] = 0.$$

To see the meaning of this condition suppose that, *hypothetically*, we observe at date t different information vectors $I_t = I_j$ with $x_t = x_j$ *all generated randomly at date t in accord with* Π then, without knowing the realizations z^j_{t+h}, we would predict ex-ante that

(6b) $$\lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^{n} [(z^j_{t+h} - E^{\Pi}(z_{t+h}|I_t = I_j))(x_j - \bar{x})] = 0 \qquad \Pi \text{ a.e.}$$

Indeed, the deduction of (6b) from (6a) is nothing but an application of the law of large numbers.

Turning to RB, let Q be a RB of an agent. The rationality conditions the RB, which are specified in terms of the time average of the conditional forecast, also imply strong orthogonality conditions. These require the conditional forecasts of z_{t+h} to be orthogonal *in the long run* to any observable x_t which has a clear and consistent meaning over time in the sense that the following time average is satisfied for Π almost all realization of the process at hand

(7a) $$\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} [(z_{t+h} - E^{Q}(z_{t+h}|I_t))(x_t - \bar{x})] = 0 \qquad \Pi \text{ a.e.}$$

However, suppose we carry out again the same experiment as above, of evaluating at some date t different information vectors $I_t = I_j$ with $x_t = x_j$ *all generated randomly in accord with* Π*.* Since $Q \neq \Pi$ we would typically conclude that

(7b) $$\lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^{n} [(z^j_{t+h} - E^{Q}(z_{t+h}|I_t = I_j))(x_j - \bar{x})] \neq 0 \quad \text{with positive } \Pi \text{ probability.}$$

That is, since the agent uses the wrong probability model, his forecast function is misspecified and hence condition (7b) says that orthogonality is likely to be violated by forecasts made at a given date or over a time interval which constitutes a fixed regime.

The difference between (7a) and (7b) is central to the empirical implications of the RB theory. Condition (7b) is a direct consequence of the fact that an RB is typically an incorrect model and if subjected at any date to a repeated sequence of hypothetical empirical tests, it would fail. It is crucial to note that probability theory guarantees that the following is true

(7b') $$\lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^{n} [(z^{j}_{t+h} - E^{Q}(z_{t+h} | I_t = I_j))(x_j - \bar{x})] = 0 \qquad Q \text{ a.e.}$$

but not (7b). Turning to condition (7a), it requires the forecasts to be correct *on average in the long run*. Kurz (1994) shows that this orthogonality condition originates in the internal structure of the reasoning used by an agent who adopts Q as a procedure of *how to deviate from the stationary forecast over time*. Although an agent may use a forecasting model which is wrong at any date, he does not make systematic mistakes over the long run.

To illustrate the reasoning above, let us apply condition (7a) to our results above. In that case the RB theory offers two predictions which we now discuss.

(i) *Prediction 1*: If we recompute the regressions (4a)-(4b) without the time periods, the averaging over time and regimes would result *in much reduced significant effects of the variables and estimates which are much closer to satisfy orthogonality*. This is so since agents do not make *consistent* mistakes over long periods: mistakes over different regimes are different and will tend to average out.

Our empirical results are in accord with prediction 1. Reporting all these results for all forecasters and for all horizons would use too much space hence we do not report all of it here. However, taking the Fed forecasts as an illustration, we estimate the following models (4a')-(4b')

(4a') $$\pi_{t,h} = \hat{\alpha}_0^{h,GB} + \hat{\alpha}_1^{h,GB}\pi_{t,h}^{GB} + \hat{\alpha}_2^{h,GB}FF_{t-1} + \hat{\alpha}_3^{h,GB}CFF_{t-1} + \hat{\varepsilon}_{t,h}^{GB}$$

(4b') $$g_{t,h} = \hat{\delta}_0^{h,GB} + \hat{\delta}_1^{h,GB}g_{t,h}^{GB} + \hat{\delta}_2^{h,GB}FF_{t-1} + \hat{\delta}_3^{h,GB}CFF_{t-1} + \hat{\upsilon}_{t,h}^{GB}.$$

It is sufficient to look at the adjusted R^2 and compare them with the results reported in Table 4. We report in Table 4A the adjusted R^2 for models (4a') - (4b') in comparison with models (2a) - (2b) and (4a') - (4b'). It is seen that the adjusted R^2 in columns (4a') and (4b') are very close to columns (2a) and (2b) and are significantly far from the results of models (4a) - (4b) which incorporated the regime variables. Indeed, Table 4A provides us with an opportunity to explain how to test for the difference between rational expectations and rational beliefs. To do that we

Table 4A: Comparing Adjusted Correlation Coefficients for Fed's Forecasts: Models (2a)-(2b),(4a)-(4b),(4a')-(4b')

		Inflation			*GNP Growth*	
Horizon h	Model (4a)	Model (4a')	Model (2a)	Model (4b)	Model (4b')	Model (2b)
0	.87	.83	.83	.54	.53	.53
1	.84	.73	.72	.32	.27	.25
2	.77	.63	.60	.23	.19	.18
3	.77	.62	.54	.17	.08	.08
4	.85	.65	.54	.25	.19	.16
5	.91	.68	.54	.22	.17	.18
6	.88	.67	.59	.26	.22	.22

use the notation $\text{model}(X) \approx \text{model}(Y)$ to mean that the parameters of models X and Y are the same. With this notation, the test criteria are as follows:

(a) *Reject rational expectations* if one of the following hypotheses is rejected:

(i) model (4a)≈ model (4a') ≈ model (2a);

(ii) model (4b)≈ model (4b') ≈ model (2b);

(iii) $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ and $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ for all h.

(a) *Reject rational beliefs* if over long time one of the following hypotheses is rejected:

(i) model (4a')≈ model (2a);

(ii) model (4b')≈ model (2b);

(ii) *Prediction 2*: If we compute models (4a') - (4b') for different time intervals, the estimates will exhibit parameter instability: the estimates will be different for different periods. Thus, if we recompute (4a')-(4b') in 2010 when we get the Fed forecasts for 2005 to cover the ten years 1995-2005 the estimates for that period will be different. The long term orthogonality of RB occurs since rational agents make only mistakes which, in the long run, cancel each other out as they arise from deviations of the forecast functions from the stationary forecasts. Rational agents do not forecast *consistently above* or *consistently below* the stationary forecasts!

The empirical evidence supports prediction (ii) as well. First, an examination of the pattern of individual private forecasts, as in Tables 5A-5B for the period 1980-2001, shows they fluctuate within the distribution of forecasters: sometimes they are at the top and sometimes they are at the bottom of the distribution. Since the distribution itself fluctuates around the stationary forecast, it follows that so do all individual forecasts. Second, we cannot recompute (4a') - (4b') to include data for 1995-2005, but we can see from Tables 7A-7C that the estimated parameters are very different for the three sub-periods under study. This supports the well established fact that forecasting models of inflation and GNP growth exhibit strong parameter instability over time (for details see Stock and Watson (1999a), (1999b), (2001)).

Applications of the RB Theory

The central idea which the RB theory offers is that in an economy with diverse and correlated beliefs, the guiding principle is that *diversity of beliefs generates volatility*. The presence of diverse beliefs expands the state space of the economy to include the state of beliefs defined by the distribution of beliefs in the market. Equilibrium quantities then depend not only on exogenous shocks, but *also upon the distribution of beliefs*. Over time, this diversity has a dual effect on economic volatility. First, it *amplifies* the impact of exogenous shocks on economic variables and second, it *generates spontaneous fluctuations* induced by the fluctuations in the state of beliefs. Kurz (1974) called the combined effect of beliefs "Endogenous Uncertainty." The basic mechanism in which RB contributes to market volatility was developed in recent years (e.g. Kurz (1994), (1997), (1998), Kurz and Schneider (1996) and Nielsen (1996) and the RB theory has been employed in several applications. For example, Kurz and Beltratti (1997) and Kurz and Motolese (2001) study the equity premium puzzle and stochastic volatility in asset markets; Wu and Guo (2001) investigate the nature of speculation in asset markets; Garmaise (1998) examines the optimal structure of a firm's capital in a market where agents hold RB; Kurz (1997a) studies the volatility of foreign exchange rates and the Forward Discount Bias in foreign exchange markets while Nielsen (1996) studies the rational for currency unions; Motolese (2000), (2001) shows money non-neutrality under RB and Kurz, Jin and Motolese (2001) examine the general properties of a monetary economy under RB.

We now return to the puzzling question of the "rationality" conditions $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ and

$\alpha_1^{h,k} = \delta_1^{h,k} = 1$. If we demonstrate -- as we have done here -- that forecast errors violate orthogonality, under what conditions could we expect $\alpha_0^{h,k} = 0$ and $\alpha_1^{h,k} = 1$ in a regression like

(1a) $$\pi_{t,h} = \alpha_0^{h,k} + \alpha_1^{h,k}\pi_{t,h}^k + \varepsilon_{t,h}^k ?$$

In the next section we show that (i) in a changing environment where agents do not know the true forecast functions and adopt non-stationary beliefs, there is no statistical way for them to guarantee satisfying $\alpha_0^{h,k} = 0$ and $\alpha_1^{h,k} = 1$ and hence, we should expect these conditions to hold only under very special conditions; and (ii) since in the specified environment $\alpha_0^{h,k} = 0$ and $\alpha_1^{h,k} = 1$ are not expected to hold, it is inappropriate to consider agents who violate these conditions as "irrational." The theory of Rational Belief explains why rational agents do not necessarily satisfy either $\alpha_0^{h,k} = 0$ and $\alpha_1^{h,k} = 1$ or orthogonality over *all* intervals of time and it also specifies the conditions under which they would satisfy them.

6.2 *Volatile Forecasts Violate* $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ *and* $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ *while the Stationary Forecasts Satisfy them*

We answer the above question by studying first a simple model where the observables are (z_{t+1}, x_t). For simplicity consider only one dimensional variables although x may, in general, be a long vector which includes lagged z variables. The true relationship is non-stationary

(8) $$z_{t+1} = \alpha_t + \beta_t x_t + \varepsilon_{t+1}$$

where the parameters constitute a dynamical system, the error has a zero mean and is uncorrelated with x_t. In most applications the parameters form a sequence of "regimes" which change *slowly relative to the rate at which observables arrive* but such that each regime generates a relatively small number of observations. Looking back in time at each regime, we sometimes could have sufficient number of observations to enable an identification of the regime's parameters with some reliability. However, in real time it is usually very difficult to identify regime parameters and "learning" within a regime, in the sense of discovering the true parameters of a regime, is limited. By the time you have sufficient data to identify a regime with high degree of reliability, a new regime would typically be in place and the old data would be of limited value.

To maintain simplicity we assume the parameters are not correlated with date t observables (z_t, x_t) or past observables and are themselves uncorrelated so that

$$\text{Cov}(\alpha, \beta) = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} (\alpha_t - \bar{\alpha})(\beta_t - \bar{\beta}) = 0.$$

This assumption is not essential and we remove it later; it simplifies the calculations below. The parameters of the system can be so volatile that the data generated have no empirical distribution. The condition of "statistical stability" which is central to the RB theory (for a precise definition, see Appendix B, Property 1), assumes that all empirical moments of the observables exist and are finite.[9] It amounts to assuming that *the structural parameters change slowly and have their own moments.* That is, we assume that for *almost all realizations* (z_{t+1}, x_t) *for t = 1, 2, ... of the stochastic process at hand*, the following exist for all integers (u, v):

(9a) $$(\bar{z}, \bar{x}) \equiv (\lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} z_{t+1}, \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} x_t)$$

(9b) $$Z^u X^v \equiv \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} (z_{t+1} - \bar{z})^u (x_t - \bar{x})^v.$$

Since parameters are not correlated with observables, we know that $\bar{z} = \bar{\alpha} + \bar{\beta}\bar{x}$.

We shall now prove that the stationary measure, denoted by m, implies the following relationship between observables

(10) $$z_{t+1} = \bar{\alpha} + \bar{\beta} x_t + \tilde{\varepsilon}_{t+1}.$$

That is, if you consider the simple model $z_{t+1} = \alpha + \beta x_t + \tilde{\varepsilon}_{t+1}$. Then, parameters estimated under the assumption the data is generated by a stationary process are $(\bar{\alpha}, \bar{\beta})$. To that end compute

$$\begin{aligned} ZX \equiv \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} (z_{t+1} - \bar{z})(x_t - \bar{x}) &= \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} (\alpha_t + \beta_t x_t + \varepsilon_{t+1} - \bar{z})(x_t - \bar{x}) \\ &= \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} ((\alpha_t - \bar{\alpha}) + \beta_t x_t - \bar{\beta}\bar{x})(x_t - \bar{x}) \\ &= \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} (\beta_t (x_t - \bar{x}) + (\beta_t - \bar{\beta})\bar{x})(x_t - \bar{x}) \\ &= \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} [\beta_t (x_t - \bar{x})^2 + \bar{x}(\beta_t - \bar{\beta})(x_t - \bar{x})] \\ &= \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \beta_t (x_t - \bar{x})^2 \end{aligned}$$

[9] The assumption of stability does not require that all data must satisfy the specified condition since observations on growth processes typically do not have finite moments. The condition means that there is some deterministic transformation, such as detrending, that leaves us with data which satisfies the condition of stability. In the example here, it will amount to the requirement that over time the sequence (α_t, β_t) fluctuates around its mean $(\bar{\alpha}, \bar{\beta})$.

$$= \bar{\beta} X^2.$$

hence

$$\bar{\beta} = \frac{ZX}{X^2}.$$

We denote the stationary forecast by $E^{m}[z_{t+1} \mid I_t] = \bar{\alpha} + \bar{\beta} x_t$.

Now let us consider any other RB which we denote by Q. For simplicity, let us assume that it implies a forecast function of the form

(11) $$E^{Q}[z_{t+1} \mid I_t] = \hat{\alpha}_t + \hat{\beta}_t x_t.$$

Rationality of belief conditions imply, among other conditions, that

(11a) $$\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \hat{\alpha}_t = \bar{\alpha} \quad , \quad \lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \hat{\beta}_t = \bar{\beta}$$

(11b) $$\mathrm{Cov}(\hat{\alpha}, \hat{\beta}) = \lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} (\hat{\alpha}_t - \bar{\alpha})(\hat{\beta}_t - \bar{\beta}) = \mathrm{Cov}(\alpha, \beta) = 0$$

Now suppose we estimate a regression like (1a) for the model at hand. We thus study the equation

(12) $$z_{t+1} = \gamma_0 + \gamma_1 E^{Q}[z_{t+1} \mid x_t] + \varsigma_{t+1} \equiv \gamma_0 + \gamma_1 [\hat{\alpha}_t + \hat{\beta}_t x_t] + \varsigma_{t+1}.$$

Are there conditions under which $\gamma_0 = 0$, $\gamma_1 = 1$? Proceeding formally we know that

$$\gamma_1 = \frac{\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} (\alpha_t + \beta_t x_t - (\bar{\alpha} + \bar{\beta}\bar{x}))(\hat{\alpha}_t + \hat{\beta}_t x_t - (\bar{\alpha} + \bar{\beta}\bar{x}))}{\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} (\hat{\alpha}_t + \hat{\beta}_t x_t - (\bar{\alpha} + \bar{\beta}\bar{x}))^2}$$

$$= \frac{\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} ((\alpha_t - \bar{\alpha}) + \beta_t (x_t - \bar{x}) + \bar{x}(\beta_t - \bar{\beta}))((\hat{\alpha}_t - \bar{\alpha}) + \hat{\beta}_t (x_t - \bar{x}) + \bar{x}(\hat{\beta}_t - \bar{\beta}))}{\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} ((\hat{\alpha}_t - \bar{\alpha}) + \hat{\beta}_t (x_t - \bar{x}) + \bar{x}(\hat{\beta}_t - \bar{\beta}))^2}.$$

It is clear that, in general, γ_1 will not equal 1. Take first the extreme case when the belief of the agent is such that there is no correlation between $(\hat{\alpha}_t, \hat{\beta}_t)$ and the true parameters (α_t, β_t) keeping in mind the assumption that $(\hat{\alpha}_t, \hat{\beta}_t)$ are not correlated with x_t. In that case

$$\gamma_1 = \frac{\bar{\beta}^2 \sigma_x^2}{\bar{\beta}^2 \sigma_x^2 + \sigma_{\hat{\beta}}^2 \sigma_x^2 + \sigma_{\hat{\alpha}}^2 + \bar{x}^2 \sigma_{\hat{\beta}}^2 + \mathrm{cov}(\hat{\alpha}, \hat{\beta})}.$$

Since $\mathrm{cov}(\hat{\alpha}, \hat{\beta}) = 0$, we have proved $0 < \gamma_1 < 1$.

Let us focus on the intuition of this result. When the structure is changing and the forecasting model is not the truth, it is impossible to ensure a regression coefficient of $\gamma_1 = 1$. Indeed, if the agent believes that the structure is changing, there is no statistical method that will *guarantee*, ex-ante, that for each value of $E^Q[z_{t+1} \mid I_t]$, the mean realization of z would be the correct one. To understand the rationality conditions on which the theory of rational belief insists, consider $E^Q[z_{t+1} \mid x_t = x] = \hat{\beta}_t x$. Here we ask what is the mean forecast, *given a specific value of the independent variable x*. Since the true model is $z_{t+1} = \alpha_t + \beta_t x_t + \varepsilon_{t+1}$, the mean value of z for each value of $x_t = x$ is $\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \beta_t x = \bar{\beta} x$. This statistical fact is known to all agents and hence the rationality of belief conditions require the forecast function to satisfy (see (11a) above)

$$\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} E^Q[z_{t+1} \mid x_t = x] = \lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \hat{\beta}_t x = [\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} \hat{\beta}_t] x = \bar{\beta} x \qquad \text{for all x.}$$

This condition says that *for any value of x the forecast of the agent must, on average, be correct.* This condition can be statistically verified.

Let us now compute the value of γ_1 under the assumption that the agent adopts, as his belief, the stationary forecast. In that case $E^m[z_{t+1} \mid I_t] = \bar{\alpha} + \bar{\beta} x_t$ and we can compute γ_1 to be

$$\gamma_1 = \frac{\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} (\alpha_t + \beta_t x_t - (\bar{\alpha} + \bar{\beta} \bar{x}))(\bar{\alpha} + \bar{\beta} x_t - (\bar{\alpha} + \bar{\beta} \bar{x}))}{\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} (\bar{\alpha} + \bar{\beta} x_t - (\bar{\alpha} + \bar{\beta} \bar{x}))^2}$$

$$= \frac{\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} ((\alpha_t - \bar{\alpha}) + \beta_t (x_t - \bar{x}) + \bar{x}(\beta_t - \bar{\beta}))(\bar{\beta}(x_t - \bar{x}))}{\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} (\bar{\beta}(x_t - \bar{x}))^2} = 1.$$

Hence, under the RB theory, to ensure $\gamma_0 = 0$, $\gamma_1 = 1$ the agent must either know the truth or adopt the stationary forecast. We have thus demonstrated that for any RB of the form written down in (11), we know that the closer the implied forecast is to the stationary forecast, the closer would the estimated parameters be to satisfy $\gamma_0 = 0$, $\gamma_1 = 1$. Moreover, the more volatile the forecasting function of the agent is, the further does he move away from satisfying this condition.

The conclusions above can be generalized far beyond the simple example provided above. More specifically, the RB theory implies the following theorem:

<u>Theorem:</u> Let $\{x_t \in X \ , \ t = 1,2,...\}$ be a stable sequence of random observables with true probability Π on the measurable space $((X^\infty),\mathcal{B}(X^\infty))$. Let m be the stationary measure on $((X^\infty),\mathcal{B}(X^\infty))$. Denote by $E^m(z_{t+h} \mid I_t)$ a one dimensional forecast of an observable variable z (a component of x) given the history $I_t = (x_1, x_2, \ldots, x_t)$ at date t. Then

(i) $E^m[E^m(z_{t+h} \mid I_t)] = \lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} E^\pi z_t = \bar{z}$ $\qquad \Pi$ a.e.

(ii) $\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} (z_{t+h} - \bar{z})(E^m(z_{t+h} \mid I_t) - \bar{z}) = \lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} (E^m(z_{t+h} \mid I_t) - \bar{z})^2$ $\quad \Pi$ a.e.

<u>Proof:</u> See Appendix B.

To sum up, from the perspective of the RB theory, if forecasts satisfy $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ and $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ then either the forecaster used the true model or that he adopted the stationary forecasts. The important implication is that in a non stationary environment, when the parameters of the system change, a forecaster who adopts the stationary forecast *would be wrong*, his forecasts will violate orthogonality in different ways over different short periods of time, but will satisfy the conditions $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ and $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ over the long run. Equally so, a forecaster who holds a rational belief and adopts a volatile forecast is very likely to violate both short term orthogonality as well as the conditions $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ and $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ at least for some horizons.

6.3 Explaining the Pattern of the "Rationality" Conditions

The analysis above provides a basic reference of the predictions of the RB theory to those circumstances when $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ and $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ hold and to the circumstances when they may be violated. To apply the above results we can then arrive at the following implications.

A. *We know that* $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ *and* $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ *are satisfied for short horizons*. Our analysis suggest that this is explained by all forecasters using forecasting models which are close to the stationary forecast for short horizons. For inflation this assumption works particularly well since inflation is very persistent hence the inflation rate at date t is a very close approximation to the inflation rate at date t +1. We have noted before that for this reason when confining attention only to the forecast of inflation over the very short horizon it is even hard to reject the assumption that agents hold rational expectations (see Keane and Runkle (1990)).

B. We know that $\alpha_0^{h,k} = \delta_0^{h,k} = 0$ *and* $\alpha_1^{h,k} = \delta_1^{h,k} = 1$ *are violated for long horizons*. To account for this it is sufficient that as the forecasting horizons gets longer, the deviations of the subjective models of agents from the stationary forecasts are larger and their non-stationary judgements play a more important role. Since we have seen that the distribution of private forecasts is very volatile, it follows that the median forecaster in each group uses a volatile forecasting function. This fact explains the strong violation of $\alpha_0^{\mathrm{h,k}} = \delta_0^{\mathrm{h,k}} = 0$ and $\alpha_1^{\mathrm{h,k}} = \delta_1^{\mathrm{h,k}} = 1$ for longer horizons.

C. *We know that (i) the Fed's forecasting model violated short term orthogonality, and (ii) we cannot reject the hypothesis* $\alpha_0^{h,GB} = \delta_0^{h,GB} = 0$ *and* $\alpha_1^{h,GB} = \delta_1^{h,GB} = 1$ *for all forecasting horizons*. The RB theory offers a direct way to reconcile these two facts. It proposes that the Fed's forecasting model was close to the conservative, stationary forecast for all horizons. This model was the wrong model since the true economic environment of the last 30 years exhibited strong non-stationarity properties. Since the model was wrong, the forecasts violate short term orthogonality. However, being the stationary forecast, on average it satisfies $\alpha_0^{\mathrm{h,GB}} = \delta_0^{\mathrm{h,GB}} = 0$ and $\alpha_1^{\mathrm{h,GB}} = \delta_1^{\mathrm{h,GB}} = 1$ over the long run.

7. Conclusions

We have presented in this paper evidence that the R&R (2000) conclusion of superior information of the Fed's staff is flawed. The logical foundations of an argument in favor of the Fed's information superiority are the *strict* orthogonality conditions of rational expectations which are violated both by the Fed's forecasts as well as by the private forecasts. We have also provided substantial evidence for the alternative perspective pointing to heterogeneity in the forecasting functions of the Fed and of private professional forecasters.

The more general message of this paper is that our dynamic economic environment is non-stationary due to changes in technology, economic institutions or economic policy and in such an environment *we should not expect orthogonality to hold over relatively short horizons.* In such an economy agents do not know the true structure of the equilibrium process and although they update their beliefs over time, in real time it is impossible for them to discover the true structure of the economy. Even if the structure changes slowly so that agents can attain confidence in their assessed parameter estimates of the economy, it is mostly too late to be useful as the environment keeps changing. The violation of short term orthogonality reported here is one more rejection of rational expectations, complementing the list of references provided in Section 1. However, we view this failure of orthogonality differently from the literature cited. Within the environment we study, the model of the single representative agent whose belief is the true equilibrium process of the economy is not more than elegant science fiction, rejected by most empirical evidence. Equally so, the Behavioral Economics literature crying "irrationality" any time orthogonality is rejected, is nothing but an attack on a straw man. Both are based on the false idealization that rational agents should know what they could not possibly know. The empirical evidence suggests that a revision of this perspective is in order.

For the environment under examination the main discipline on the beliefs of agents must be based on the long term regularity of past empirical record, not on abstract knowledge of the structure which is unavailable to anyone. Based on this we suggest that the theory of Rational Beliefs provides all the restrictions on beliefs which can be expected to hold. We have also shown that the empirical evidence is compatible with the predictions of the RB theory.

Why are these conclusions important? The problem at hand is not only how to determine if forecasters are "rational" but mostly, to explain the observed pattern of market volatility. Rational expectations based models fail to account for significant components of the observed volatility.

Consequently, many dynamic phenomena have been regarded as "puzzles" or "anomalies" both in the real economy as well as in financial markets. The challenge is then to develop a unified theory of fluctuations which explains the volatility of the real economy as well as its financial markets. The RB theory offers a unified perspective by proposing that the source of most volatility, real and financial, is the *structure of agents' expectations*. The main insight of the RB theory is that in an economy with diverse and correlated beliefs, the guiding principle is: *diversity of beliefs generates volatility*. The endogenous expansion of the states space of the economy to include the state of beliefs introduces a powerful mechanism to explain the pattern of observed market volatility.

With the above perspective in mind, the importance of the results of this paper is that they provide strong evidence in support of the heterogenous forecasting paradigm. Hence, they bolster the view that economic fluctuations and market volatility have a crucial component generated by the agents' beliefs and expectations. We also argues in this paper that "rationality" of beliefs should not be defined by requiring agents to know something which they cannot know but rather, that rationality should be defined relative to the empirical evidence available to agents. The empirical results of this paper are consistent with this perspective.

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APPENDIX

APPEDIX A: Test of GNP Growth Forecast Error Orthogonality

In this appendix we present the estimates of the parameters of the model (5b) in the text

$$g_{t,h} - g^{k}_{t,h} = \sum_{j=1}^{3} 1_j(t)\left[\hat{\alpha}^{h,k}_{0,j} + \hat{\alpha}^{h,k}_{1,j} g^{k}_{t,h} + \hat{\alpha}^{h,k}_{2,j} FF_{t-1} + \hat{\alpha}^{h,k}_{3,j} CFF_{t-1}\right] + \hat{\upsilon}^{h,k}_{t,h} \tag{5b}$$

for the Federal Reserve, SPF and BLU GNP Growth forecast errors. The Fed's forecasts are for 1965:11-1995:11, SPF are for 1981:7-1995:11 and BLU are for 1980:1-1995:11. Hence, the estimates for the Fed's equations cover all three monetary regimes while for the SPF and BLU we present estimates for forecast errors over the last two regime only. The pattern of forecast errors can be examined in the same way we examined the inflation forecast errors in the text.

Table 7D: Test of Fed's GNP Growth Forecast Error Orthogonality with Monetary Policy Variables

(standard errors in parantheses)

Horizon →	0	1	2	3	4	5	6
$\hat{\delta}^{h,GB}_{0,1}$	1.80	4.35*	4.18	2.74	-8.24	-16.4*	-21.7*
	(2.63)	(2.39)	(2.75)	(4.01)	(5.82)	(5.47)	(8.09)
$\hat{\delta}^{h,GB}_{0,2}$	.07	8.49*	8.29*	5.99*	4.82	.17	-.44
	(1.17)	(2.67)	(3.08)	(2.75)	(3.32)	(3.01)	(3.20)
$\hat{\delta}^{h,GB}_{0,3}$	1.57	3.52*	3.68*	5.04*	3.49	3.52	4.26
	(.99)	(1.43)	(1.67)	(1.82)	(2.43)	(2.84)	(3.22)
$\hat{\delta}^{h,GB}_{1,1}$	-.12	-.40*	-.43*	-.51	-.78	1.93*	2.58
	(.22)	(.18)	(.23)	(.40)	(.78)	(1.02)	(1.74)
$\hat{\delta}^{h,GB}_{1,2}$	.03	-.78*	-.90*	-.76	-.36	.50	.54
	(.11)	(.38)	(.51)	(.56)	(.73)	(.77)	(.71)
$\hat{\delta}^{h,GB}_{1,3}$	-.10	-.38	-.18	-.37	-.01	-.09	-.12
	(.13)	(.31)	(.37)	(.60)	(.74)	(.86)	(.88)
$\hat{\delta}^{h,GB}_{2,1}$	-.17	-.46*	-.44	-.22	.70	1.25*	1.76*
	(.32)	(.29)	(.34)	(.43)	(.52)	(.45)	(.51)
$\hat{\delta}^{h,GB}_{2,2}$	.10	-.57*	-.50*	-.31*	-.29*	-.08*	-.03*
	(.11)	(.19)	(.24)	(.17)	(.21)	(.18)	(.18)
$\hat{\delta}^{h,GB}_{2,3}$	-.11	-.36*	-.44*	-.57*	-.45*	-.52*	-.57*
	(.14)	(.20)	(.23)	(.25)	(.27)	(.25)	(.26)
$\hat{\delta}^{h,GB}_{3,1}$	-.16	-.11	-.08	-.36*	-.41*	-.15	.40
	(.21)	(.14)	(.18)	(.19)	(.18)	(.24)	(.47)
$\hat{\delta}^{h,GB}_{3,2}$	-.03	.02	- .05	.21	- .07	.11	- .06
	(.14)	(.17)	(.18)	(.23)	(.12)	(.27)	(.18)
$\hat{\delta}^{h,GB}_{3,3}$	.06	.13	.20*	.22	.24	.22*	.17
	(.09)	(.12)	(.13)	(.15)	(.16)	(.12)	(.13)
R^2	.04	.12	.08	.12	.10	.05	.07
N	293	277	255	238	208	149	89

Table 7E:Test of Median GNP Growth SPF Forecast Error Orthogonality with Monetary Policy Variables

(standard errors in parantheses)

Horizon →	0	1	2	3	4
$\hat{\delta}^{h,SPF}_{0,2}$	-2.02	-.17	3.36	6.03*	4.48
	(2.18)	(2.47)	(2.31)	(3.03)	(3.17)
$\hat{\delta}^{h,SPF}_{0,3}$	2.56*	4.30*	2.56*	2.56*	2.56*
	(1.62)	(2.26)	(1.62)	(1.62)	(1.62)
$\hat{\delta}^{h,SPF}_{1,2}$	.46*	1.39*	.93*	-1.04	-.72
	(.24)	(.43)	(.76)	(1.04)	(.63)
$\hat{\delta}^{h,SPF}_{1,3}$	-.09	-.32	-1.02*	-1.30*	-1.36*
	(.26)	(.43)	(.48)	(.43)	(.57)
$\hat{\delta}^{h,SPF}_{2,2}$	.18	-.44*	-.69*	-.19	-.13
	(.20)	(.14)	(.18)	(.23)	(.30)
$\hat{\delta}^{h,SPF}_{2,3}$	-.21	-.43*	-.02	-.70*	-.71*
	(.20)	(.27)	(.48)	(.31)	(.28)
$\hat{\delta}^{h,SPF}_{3,2}$	.12	-.56*	-.41*	-.27	-.23
	(.16)	(.30)	(.25)	(.32)	(.34)
$\hat{\delta}^{h,SPF}_{3,3}$	.13	.19	-.66*	.20	.24*
	(.13)	(.14)	(.31)	(.14)	(.15)
R^2	.01	.29	.27	.13	.11
N	78	77	76	75	74

Table 7B:Test of Median BLU GNP Growth Forecast Error Orthogonality with Monetary Policy Variables

(standard errors in parantheses)

Horizon →	0	1	2	3	4	5	6
$\hat{\delta}^{h,BLU}_{0,2}$	1.14	6.68*	9.73*	9.45*	6.28*	5.65*	4.42*
	(1.60)	(2.60)	(2.72)	(2.59)	(1.94)	(2.28)	(2.75)
$\hat{\delta}^{h,BLU}_{0,3}$	2.88*	4.12*	8.05*	12.17*	10.39*	9.59*	10.64*
	(1.36)	(1.91)	(2.35)	(2.30)	(2.24)	(2.92)	(3.49)
$\hat{\delta}^{h,BLU}_{1,2}$	.06	-.40	-1.27*	-1.48*	-.83	-1.29*	-.43
	(.18)	(.38)	(.56)	(.76)	(.76)	(.68)	(.83)
$\hat{\delta}^{h,k}_{1,3}$	-.08	-.25	-1.23*	-2.48*	-2.05*	-1.66*	-2.25*
	(.27)	(.39)	(.50)	(.59)	(.62)	(.89)	(1.22)
$\hat{\delta}^{h,BLU}_{2,2}$	-.04	-.50*	-.51*	-.40*	-.35	-.10	-.23
	(.15)	(.19)	(.19)	(.21)	(.23)	(.23)	(.28)
$\hat{\delta}^{h,GB}_{2,3}$	-.27*	-.41*	-.69*	-.90*	-.77*	-.81*	-.81*
	(.16)	(.24)	(.29)	(.26)	(.26)	(.26)	(.27)
$\hat{\delta}^{h,BLU}_{3,2}$	.17	-.01	-.06	.14	-.32*	.00	-.05
	(.17)	(.17)	(.15)	(.18)	(.18)	(.22)	(.16)
$\hat{\delta}^{h,BLU}_{3,3}$	.15	.21*	.23*	.15	.15	.12	-.03
	(.11)	(.12)	(.13)	(.13)	(.13)	(.13)	(.17)
R^2	.02	.13	.25	.27	.28	.15	.18
N	252	249	246	243	235	174	114

Appendix B: Rational Beliefs-A Brief Summary of the Theory

The theory of Rational Beliefs (RB) starts with the observation that the true probability of observed variables is not known; agents may assume the true process is non-stationary. Lacking knowledge, rational agents develop their own theories in order to form probability beliefs about future economic events. The second assumption is that at each date t an agent has a great deal of data about past performance of the economy hence his reference is the *empirical distribution* derived from the frequency at which events occurred in the past. The third component of the theory is the observation that the economic life of an agent is short relative to the clock at which new data arrives. Hence, let $X \subseteq \mathbb{R}^N$ be the space of observables, $x_t \in X$ a vector of the N observables and let $x = (x_0, x_1, x_2, ..)$ be the random data from 0 to infinity. The history from t on is $x^t = (x_t, x_{t+1}, x_{t+2}, ...)$, hence $x^0 = x$. The history up to t is $I_t = (x_0, x_1, x_2, ..., x_t)$ and date t is *large* so that the empirical distribution of the process can be discovered. The agent's life L is the span when he makes decisions and L is *very short* relative to t. An agent's belief may be correct or not but the little data - $(x_t, x_{t+1}, x_{t+2}, ..., x_{t+L})$ - generated during his own economic life is too small to provide a *reliable* test of his theory. Even if these limited observations could negate his theory with some confidence, such evidence becomes available too late, after most economic decisions have been made. Thus, the rationality of a belief Q cannot be judged the compatibility of a belief Q *with the limit of the data in the future*. Instead, the RB theory defines rationality in terms of *its compatibility with the empirical distribution of past data.* We now explain the rationality conditions of the theory.

Let X^∞ be the space of infinite sequences x and $\mathcal{B}(X^\infty)$ be the Borel σ-field of X^∞. Let Π be the true probability on $(X^\infty, \mathcal{B}(X^\infty))$ under which the data is generated; this process may be non-stationary. For each finite dimensional set $B \in \mathcal{B}(X^\infty)$ agents compute the empirical frequency of the event defined by

$$m_n(B)(x) = \frac{1}{n} \sum_{k=0}^{n-1} 1_B(x^k) = \left\{ \begin{array}{l} \text{The relative frequency that B occurred} \\ \text{among n observations since date 0} \end{array} \right\}$$

where

$$1_B(y) = \begin{cases} 1 & \text{if } y \in B \\ 0 & \text{if } y \notin B \end{cases}.$$

The RB theory assumes the data generating process is *stable* in the sense that the limits above exist Π a.e. This is Property 1. It implies that an empirical distribution exists and agents learn from it a probability m on $(X^\infty, \mathcal{B}(X^\infty))$. We show that (i) m is unique; (ii) m is stationary and hence is called "*the stationary measure of Π*." Since m is learned from the data, there is no disagreement among agents about it. The probability m is their *common* empirical knowledge.

Agents who do not know the true probability Π discover a probability m induced by Π. If an economy is stationary, m = Π but agents could not know this fact. What are the restrictions which the knowledge of m places on the beliefs of rational agents? To explore this we introduce

Definition 1: (Property 2) A process $\{x_t, t = 1, 2, 3, ...\}$ with probability Π on $(X^\infty, \mathcal{B}(X^\infty))$ is said to be Weak Asymptotic Mean Stationary- WAMS - if for each finite dimensional event $B \in \mathcal{B}(X^\infty)$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \Pi(B^{(k)}) = \mathring{m}^{\Pi}(B) \quad \text{exists.}$$

We show that the collection $\mathring{m}^{\Pi}(B)$ induces a unique probability m^{Π} on $(X^\infty, \mathcal{B}(X^\infty))$ which is stationary. Since this holds for *any* WAMS probability Q, we denote by m^Q the probability on $(X^\infty, \mathcal{B}(X^\infty))$ induced by the above analytical property which Q satisfies. The central result of the RB theory is the following:

Theorem 1: Properties 1 and 2 are equivalent and $m(A) = m^{\Pi}(A)$ for *all events* $A \in \mathcal{B}(X^\infty)$.

Agents compute m from the data and Theorem 1 leads to a natural definition of what it means for a probability belief Q to be "compatible with the data", which m represents:

Definition 2: A probability belief Q is said to be *compatible with the observed data m* if

(i) Q is a WAMS probability on $(X^\infty, \mathcal{B}(X^\infty))$,
(ii) $m^Q(A) = m(A)$ for all events $A \in \mathcal{B}(X^\infty)$.

Condition (ii) is the main implication. Now consider a rational agent with a stable belief Q. He knows m^Q by the analytic conditions above. If $m^Q \neq m$, it would prove that Q is not the truth. Indeed, it would prove that Q is not compatible with the data m. This proposes a natural definition of an RB:

Definition 3: A probability belief Q on $(X^\infty, \mathcal{B}(X^\infty))$, is said to be *a Rational Belief (RB) relative to m* if Q is compatible with the known data m.

An agent who holds a rational belief Q relative to m must then satisfy the rationality conditions

$$(\star) \qquad m^Q(B) = m(B) \quad \text{for all sets } B \in \mathcal{B}(X^\infty).$$

These are the central restrictions of the RB theory.

The implication of (★) is that under RB agents may disagree about probabilities of short term events but not about long term averages. Note that (i) Π is a Rational Belief and hence REE is an RBE; (ii) m is an RB although it is possible that $m \neq \Pi$; (iii) an RB Q and the true Π may disagree on timing or sequencing; (iv) an RB Q and the true Π may put different probabilities on important rare events; (v) an RB Q allows optimism/pessimism relative to m.

Three Observations

(i) If agents believe the economy is stationary then $Q^k = m$. Thus, in an RBE agents may believe the economy is non-stationary and disagree about *unknown* regime parameters (e.g. mean value function) of the process.

(ii) When holding diverse belief agents' beliefs affects demands and equilibrium maps have the general form

$$p_t = \Phi(I_t, Q_t^1, Q_t^2, \ldots, Q_t^H)$$

where $(Q_t^1, Q_t^2, \ldots, Q_t^H)$ are date t conditional probabilities. In such equilibria beliefs act as a propagation mechanism of economic fluctuations. Hence, the RBE theory proposes that economic fluctuations have an endogenous component propagated *within* the economy. Kurz [1974] called it *Endogenous Uncertainty*.

(iii) *Diversity Implies Volatility*: *disagreement among rational agents implies their conditional probabilities fluctuate over time.* For example, consider a finite state Markov economy. A belief Q^k is represented by a sequence of selections from, say, 2 different Markov matrices $\{G_1^k, G_2^k\}$ which k believes are possible. m is represented by a *single* matrix Γ. If agents disagree, $Q^k \neq m$ imply that $G_j^k \neq \Gamma$ for j = 1,2. Rationality requires the time average of the G_j^k used by k to equal Γ. Agent k would be irrational if he used only one matrix, say $G_1^k \neq \Gamma$, since the time average under G_1^k is not Γ. When rational agents disagree they must use *varying* matrices over time.

The main problem now is the simplification of the rationality conditions ($\star$). Since agents hold diverse non-stationary beliefs, the problem is to find a simple mathematical tool for describing non-stationarity. The tool developed is the Conditional Stability Theorem (Kurz and Schneider [1996] and Nielsen [1996]). To explain this theorem note that we want to describe in a tractable way the non-stationarity of a process on $((X)^\infty, \mathcal{B}((X)^\infty)$. The Conditional Stability Theorem describes all the non-stationarity using artificial variables $y_t \in Y$ with a marginal probability space $((Y)^\infty, \mathcal{B}((Y)^\infty, \mu)$. It postulates $(X \times Y)$ to be the state space, defines a universal probability Q on the space $((X \times Y)^\infty, \mathcal{B}((X \times Y)^\infty)$ and defines the desired *non-stationary* probability to be Q_y, the *conditional* probability of Q with respect to the sequence $y = (y_1, y_2, \ldots)$. Thus Q must satisfy the condition that for all $A \in \mathcal{B}(X^\infty)$ and $B \in \mathcal{B}(Y^\infty)$

$$Q(A \times B) = \int_B Q_y(A)\, \mu(dy).$$

The conditional probability space $((X)^\infty, \mathcal{B}((X)^\infty, Q_y)$ is non-stationary since probabilities of events in $\mathcal{B}((X)^\infty)$ are not time independent: they change with the parameters y_t which are time dependent. If Q_y reflect the non-stationarity of the system, then we interpret $\{y_t, t = 1, 2, \ldots\}$ as a *mathematical description* of that non-stationarity. This approach is commony used where Y is the set of possible "regimes" (finite or infinite) and y_t identifies the regime at t. Here we use this method to describe the perceived non stationarity of an agent. Thus, $Q^k_{y_t}$ is the date t probability belief of future observables by k and y_t^k reflects the "*perception*" or "*assessment*" of agent k. The assessment variables $\{y_t^k, t = 1, 2, \ldots\}$ are generated by the agent himself, providing a vocabulary to describe his belief in the non-stationarity of the observables. From an informational perspective, assessment variables are privately perceived parameters indicating how an agent interprets current information. These purely subjective variables should not be taken to be objective and transferable "information". Since even within an infinite horizon model each family member lives for finite number of periods, at t he does not know y_τ^k for dates τ not in his own lifetime. In case of n agents the vector of assessment variables $(y_t^1 \ldots y_t^n)$ may be correlated and jointly distributed with observables. Agents do not know these distributions and cannot learn them from data since k "knows" only his own parameter y_t^k.

This approach leads to a technical difficulty. To be compatible with the earlier development, how do we know when a *conditional* system $((X)^\infty, \mathcal{B}((X)^\infty, Q_y^k)$ is stable and Properties 1 and 2 satisfied? And how do we compute the stationary measure of such a conditional system? For answers, note that since the conditional system is intended to describe *all* the non-stationarity, we can assume the *joint* system to be stationary. Now, for any joint system $((X \times Y)^\infty, \mathcal{B}((X \times Y)^\infty), Q)$, the marginal measure Q_{X^∞} is defined by

$$Q_{X^\infty}(A) = Q(A \times Y^\infty) = \int_{Y^\infty} Q_y(A)\, \mu(dy) \quad \text{for all } A \in \mathcal{B}(X^\infty).$$

Theorem 2: (Conditional Stability Theorem, Kurz and Schneider [1996] Theorem 2) Let $((X\times Y)^\infty, \mathcal{B}((X\times Y)^\infty), Q)$ be stationary and ergodic and let Y be countable then

(a) $(X^\infty, \mathcal{B}(X^\infty), Q_y, T)$ is stable and ergodic for Q a.a. y;

(b) The stationary measure of Q_y is independent of y for Q a.a. y and if we denote it by m^{Q_y} then it satisfies the condition $m^{Q_y} = Q_{X^\infty}$.

Consider an example which we use later when $y_t^k \in Y = \{0, 1\}$, Q^k is a probability belief on the *joint* process $\{(x_t, y_t^k), t = 1, 2, ...\}$ which is a Markov process and, conditionally on y^k_{t-1}, the distribution of y_t^k is independent of other observables. Then the *effective belief* $Q^k_{y^k}$ is defined by two transition functions F_1^k and F_2^k as follows

$$Q_{y_t^k} = \begin{cases} F_1^k & \text{if} \quad y_t^k = 1 \\ F_2^k & \text{if} \quad y_t^k = 0. \end{cases}$$

The *marginal* measure $Q^k_{X^\infty}$ is the probability of a stationary Markov process uniquely defined by a transition function F^k, computed by the simple expression (which we use again later):

$$F^k = F_1^k \mu^k(y_t^k = 1) + F_2^k \mu^k(y_t^k = 0).$$

Kurz-Motolese [2001, Section 2] provides a detailed and formal definition of a Rational Belief Equilibrium (RBE) and compares it to other equilibrium concepts.

Forecasting Theorem. In the text we stated a theorem regarding the orthogonality conditions under the stationary forecast and the proof is then presented here. We then have:

Theorem 3: Let $\{x_t \in X \ , \ t = 1,2,...\}$ be a stable sequence of random observables with true probability Π on the measurable space $((X^\infty), \mathcal{B}(X^\infty))$ and with a stationary measure m on $((X^\infty), \mathcal{B}(X^\infty))$. Denote by $E^m(z_{t+h} \mid I_t)$ the one dimensional forecast of an observable variable z (a component of x) given the history $I_t = (x_1, x_2, \ldots, x_t)$ at date t. Then

$$\text{(i)} \quad E^m[E^m(z_{t+h} \mid I_t)] = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} E^{\Pi} z_t = \bar{z} \qquad \Pi \text{ a.e.,}$$

$$\text{(ii)} \quad \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} (z_{t+h} - \bar{z})(E^m(z_{t+h} \mid I_t) - \bar{z}) = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} (E^m(z_{t+h} \mid I_t) - \bar{z})^2 \quad \Pi \text{ a.e.}$$

Proof: The conclusion $E^m[E^m(z_{t+h} \mid I_t)] = E^m z_{t+h} = E^m z_\tau$ for all τ follows from the fact that m is stationary. The conclusion $\lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} z_t = E^m z_\tau$, Π a.e. for all τ follows from the construction of the stationary measure. Finally the conclusion $\lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} z_t = \bar{z}$, Π a.e. follows from stability of the underlying process $\{x_t \in X \ , \ t = 1,2,...\}$. Hence $E^m[E^m(z_{t+h} \mid I_t)] = E^m z = \bar{z}$. It then follows that $\lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} E^m(z_{t+h} \mid I_t) = E^m z_\tau = \bar{z}^m = \bar{z}$ Π a.e. Now, by assumption $z_{t+h} = \alpha + \beta E^m(z_{t+h} \mid I_t) + \varepsilon_{t+h}$, it follows from the above that $\bar{z} = \alpha + \beta \bar{z}^m$. To complete the proof of the theorem we need to show $\beta = 1$. To do that we compute the following, for Π a.e.

$$\lim_{n\to\infty}\frac{1}{n}\sum_{t=1}^{n}(z_{t+h}-\bar{z})(E^{m}(z_{t+h}|I_t)-\bar{z}) = \lim_{n\to\infty}\frac{1}{n}\sum_{t=1}^{n}(z_{t+h}-E^{m}(z_{t+h}|I_t)+E^{m}(z_{t+h}|I_t)+\bar{z})(E^{m}(z_{t+h}|I_t)-\bar{z})$$
$$= \lim_{n\to\infty}\frac{1}{n}\sum_{t=1}^{n}(z_{t+h}-E^{m}(z_{t+h}|I_t))(E^{m}(z_{t+h}|I_t)-\bar{z}) + \lim_{n\to\infty}\frac{1}{n}\sum_{t=1}^{n}(E^{m}(z_{t+h}|I_t)-\bar{z})^2.$$

We now claim that the last limit is equal to the following limit

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{t=1}^{n}(E^{m}(z_{t+h}|I_t)-\bar{z})^2.$$

To see this note that since m is stationary it follows from the Ergodic theorem that

$$\lim_{n\to\infty}\frac{1}{n}\sum_{t=1}^{n}(z_{t+h}-E^{m}(z_{t+h}|I_t))(E^{m}(z_{t+h}|I_t)-\bar{z}) \;=\; E^{m}(z_{t+h}-E^{m}(z_{t+h}|I_t))(E^{m}(z_{t+h}|I_t)-\bar{z}) \;=\; 0 \quad m \text{ a.e.}$$

However, the construction of the stationary measure ensures that each such limit is true Π a.e. Hence we have shown that $\beta = 1$ and hence $\alpha = 0$.

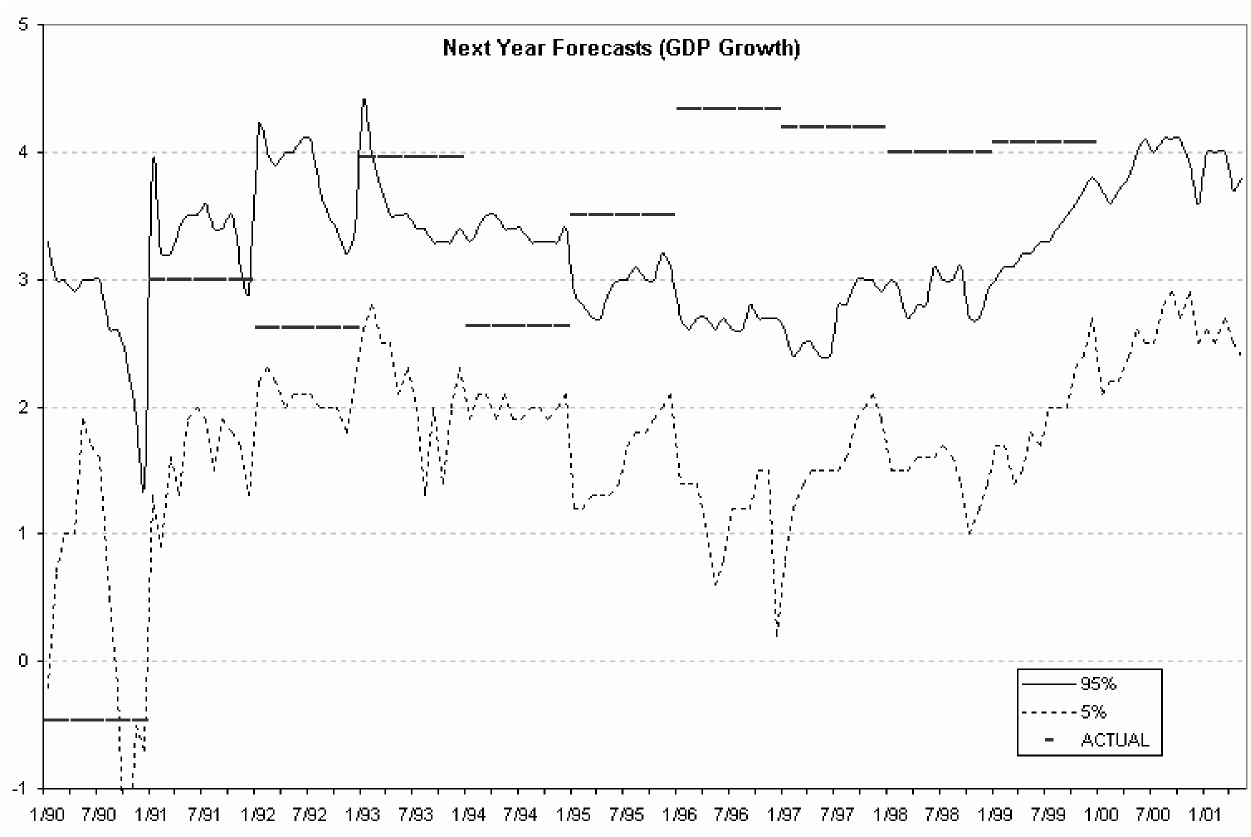

Figure 1: Distribution of Private Forcasts of GDP Growth

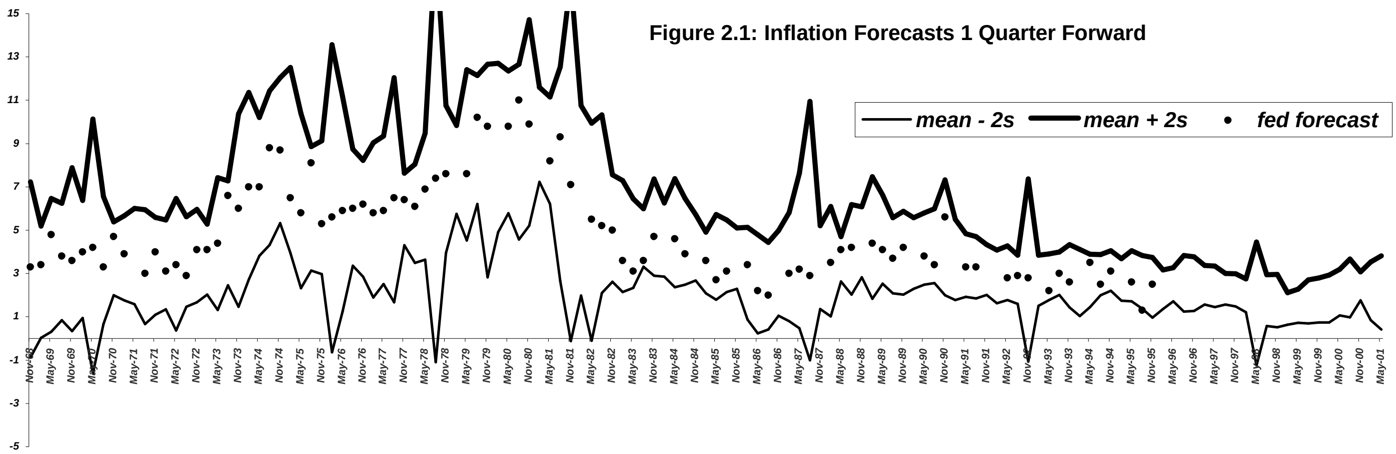

Figure 2.1: Inflation Forecasts 1 Quarter Forward

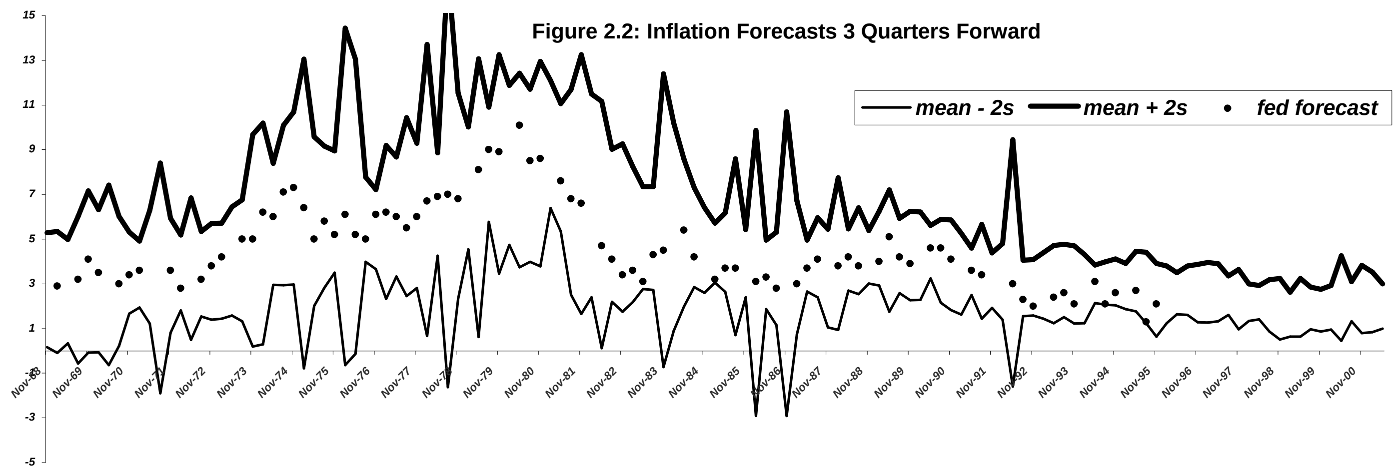

Figure 2.2: Inflation Forecasts 3 Quarters Forward

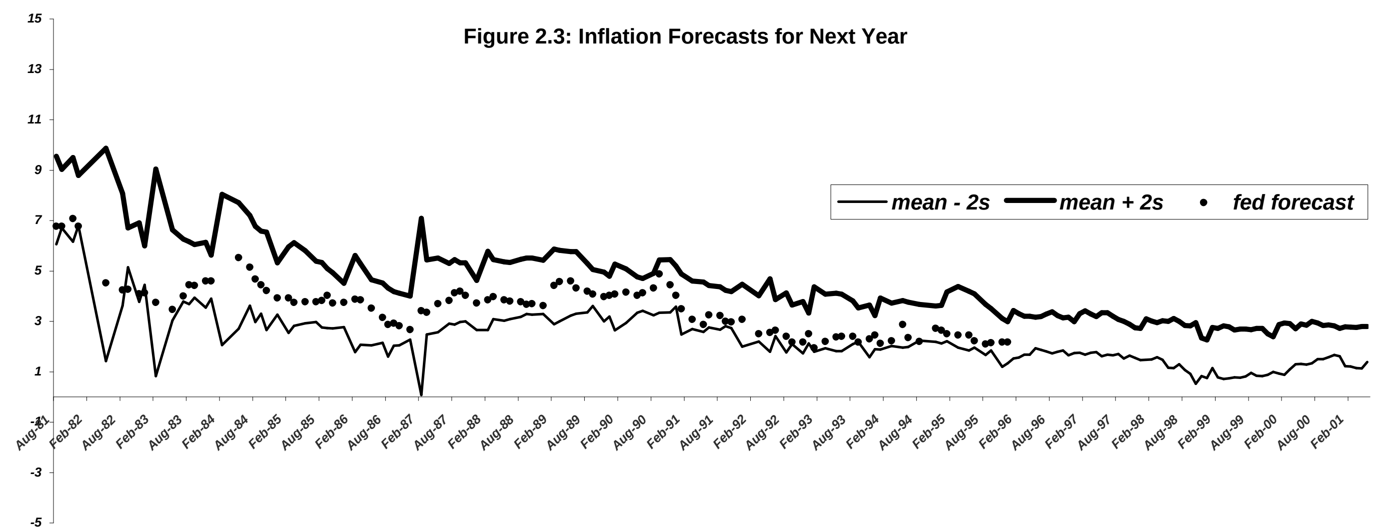

Figure 2.3: Inflation Forecasts for Next Year

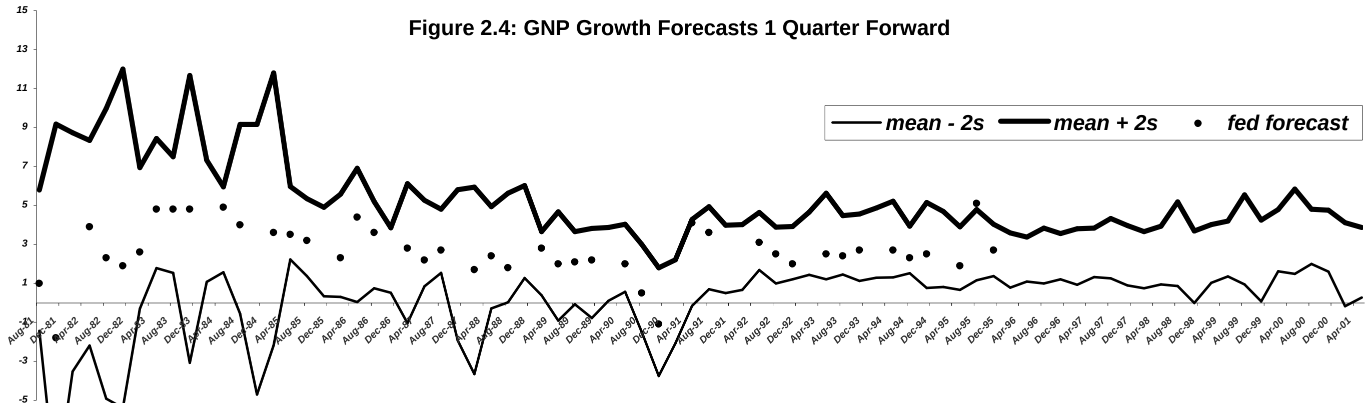

Figure 2.4: GNP Growth Forecasts 1 Quarter Forward

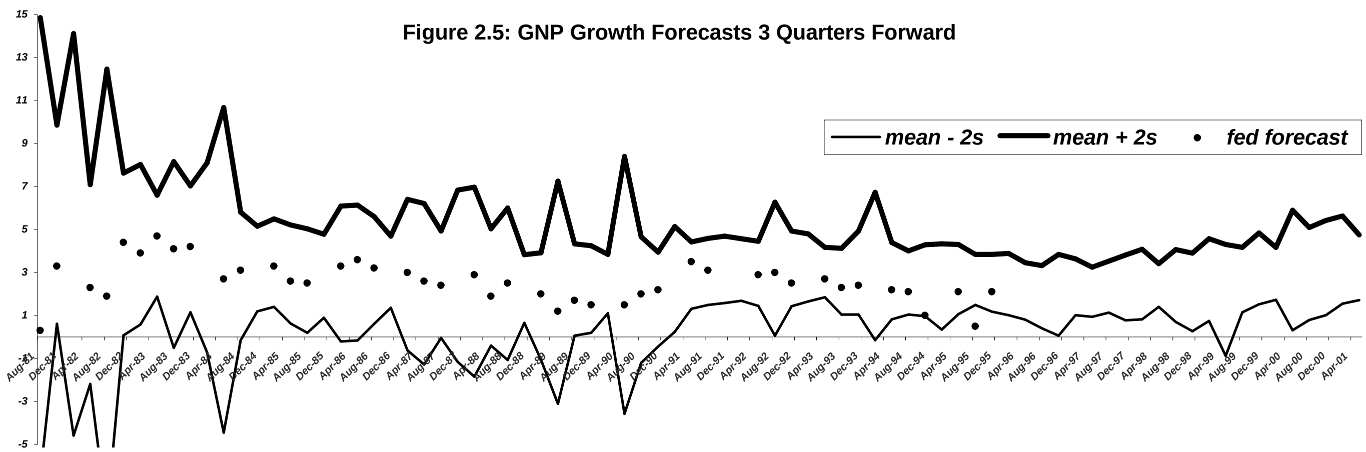

Figure 2.5: GNP Growth Forecasts 3 Quarters Forward

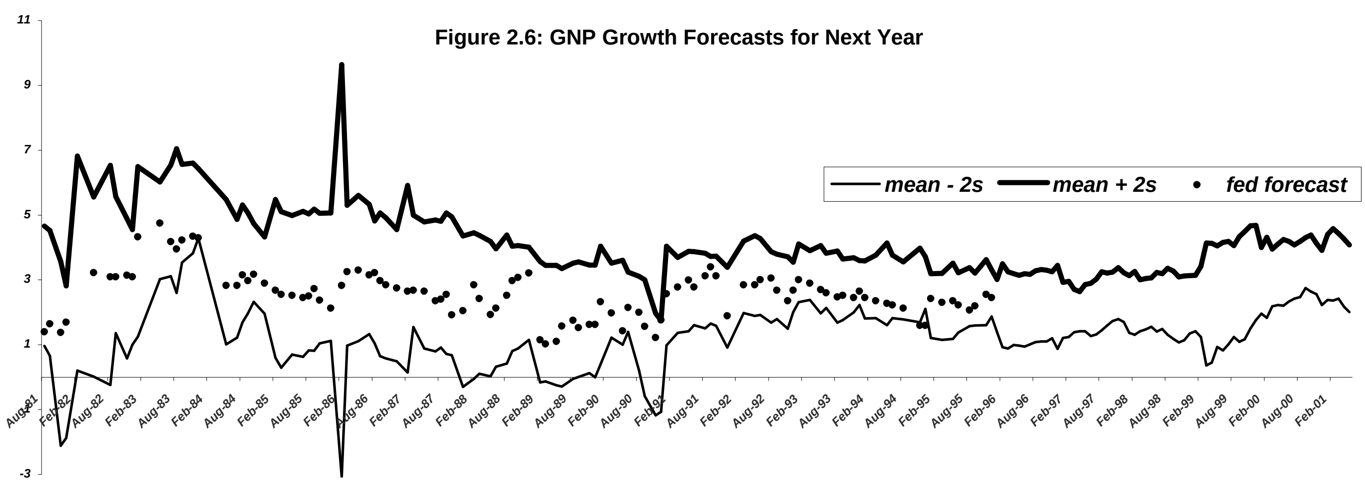

Figure 2.6: GNP Growth Forecasts for Next Year